Observing Students' Mathematical Thinking Processes and Collaboration through On-line Windows

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The purposes of this study were to examine the ways in which students develop or demonstrate their understanding of mathematical processes by utilizing technology and to evaluate the nature of their on-line collaboration. We designed and implemented a blended mathematics course for undergraduate students who are preparing to become elementary school teachers. In order to evaluate student understanding of mathematical processes and student interactivity, we examined transcripts of on-line discussions, as well as computer-based projects. Findings on two selected groups demonstrate a broad spectrum of conceptual episodes, ranging from misconceptions to "eureka" experiences, as well as significant variation in collaboration between the two groups.

Introduction

Five years ago, the National Council of Teachers of Mathematics (NCTM, 2000) articulated a vision of mathematics instruction that fosters learning with understanding by promoting mathematics processes: problem solving, reasoning and proof, communication, connections, and representation. Unfortunately, examinations of classroom lessons and teacher readiness reveal that much work needs to be done before this vision can be realized.

Based on the Third International Mathematics and Science Study (TIMMS, 1999), US students spent far less time in problem solving and reasoning activities, compared with Japanese students. Also, eighth-grade mathematics lessons in the United States were among the least likely to emphasize mathematical connections or relationships. Moreover, American teachers find it difficult to create learning environments that support students' construction of their own understanding of mathematics (Fennema and Nelson, 1997, Frykholm, 2003).

By utilizing technology, teachers can create environments in which students can learn by doing, receive feedback, continually refine their understanding, and build new knowledge (Barron et al., 1998; Becker, 2000; Jonassen, 2000; Kaput, 1992). Also, the NCTM identified the "Technology Principle" as one of six principles of high quality mathematics education (NCTM, 2000): "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24).

The use of technology itself does not necessarily lead to classrooms that focus more on mathematical processes, however. Rather, the various ways students utilize technology likely have major impact on their understandings (Bransford et. al, 1999). If so, how do we, teachers and researchers, evaluate their students' mathematical understandings? Online windows provided us with a lens to examine student use of technology and development of mathematical understandings. In this paper, we tell the stories of two specific groups.

The purposes of this study are two-fold. First, we examined the ways in which pre-service elementary school teachers develop or demonstrate their understanding of these mathematical processes. This examination is intended to address the following research questions:

- What kinds of understandings of mathematical processes do these students develop and demonstrate in a technologically-enriched course?
- To what extent do they utilize available technology in order to demonstrate these understandings?

Second, we evaluated the nature of their on-line collaboration as a means to developing these understandings, in order to answer the following questions:

- How do these student groups function in an on-line environment?
- How do they use an on-line environment, in conjunction with other communication channels?

By addressing these research questions, we intend to speak to a more general issue: To what extent does a technologically-enriched class allow clear observation of the development of student understandings of mathematical processes and collaboration?

Method and Procedure

The Course

Using a new course development model (Cerreto and Lee, 2004), we designed and implemented a blended mathematics course, Elementary School Mathematics: Numbers and Patterns, for prospective elementary school teachers. The course was offered at a mid-size, public, liberal arts college, located in northeastern United States. At least one section of the course was taught by one of us each semester from Spring 2003 to Spring 2005.

The class format included small group work and whole-class discussions, concerning mathematical concepts and their connection to the classroom. Students worked outside of class in groups of three to four to complete three projects during the semester. Each project consisted of a series of explorations on a particular mathematical topic (Bassarear, 2001). Groups presented their work on the projects to their classmates during three in-class showcases held throughout the semester.

In this blended course, computer technology was used to cultivate mathematical processes. All class sessions were held in either a computer lab or an electronic classroom, equipped with Internet access, Microsoft Office suite, as well as other software. Students and the instructor used an on-line environment, WebCT, to facilitate communication

Technology served many purposes in this course. In the classroom, the instructor used presentation software and Internet resources as tools for the discovery and discussion of mathematical content. The instructor also modeled the appropriate use of technology to promote mathematical learning. All lessons were presented using slides shows, many utilizing animation to illustrate mathematics ideas. Typical class sessions also included examples of ways to use spreadsheets to explore mathematical ideas as well as visits to Websites containing mathematical activities.

Outside of class, WebCT served as the communication hub of the course. This environment provided a platform for instructor-student and student-student communication, as well as weekly discussion threads, in which the instructor and students were able to share their thoughts on the weekly agenda. Finally, private, small group conferences served as a virtual meeting place for students to discuss and share their work on each of three projects during the course.

The instructor provided feedback to groups through these on-line conferences. After each group submitted each project draft online, the instructor responded with acknowledgement of areas of progress and suggestions for improvement. After each group submitted its final version, the instructor provided a summative evaluation and assigned a project grade to the group. In addition, the instructor monitored each group conference regularly and occasionally posted comments.

Students were expected and encouraged to utilize these small group on-line conferences as they worked on their projects, but they were not given specific rules defining acceptable participation. Consequently, the ways in which students used these on-line conferences varied greatly. Some communicated solely through the on-line conferences; others used a combination of the on-line conferences, other instant messaging systems, and telephone and/or face-to-face meetings; and a few groups held all of their meetings in person and used the conference only to summarize their meetings (at the instructor's request) and submit their projects. Of those who used the conferences for discussion, a few groups established on-line meeting times and held their discussions in real time. However, most groups communicated asynchronously.

Subjects

The subjects of this study consist of a total of 61 students enrolled in three sections of the course, one section taught during the Fall 2004 semester and two during the Spring 2005 semester. As in previous semesters, virtually all of the students were taking this course in order to fulfill the requirements of the teacher certification program. Most of these students did not bring with them strong mathematical backgrounds; in fact many of the students who take this course harbor negative attitudes about mathematics and mathematics teaching and learning (Cerreto and Lee, 2004). Although a few mathematics majors have enrolled in the course over the past semesters, the course is usually populated by students in non-mathematics-intensive majors.

Data

All on-line discussions resulted in permanent transcripts that could be reviewed at a later time. For this study, we examined transcripts from all 54 small group conferences for evidence of development of the process standards and intra-group collaboration. This paper focuses on the data collected from one project for each of two specific groups, selected because they appeared to represent an interesting paradox between product and process. Both groups produced projects that were of very good quality. However, one group (Group A) demonstrated superior collaboration, while the other (Group B) showed serious problems in this area. The decision to select extreme cases for discussion is consistent with generally accepted practice (e., g. Frykholm, 2003).

Analysis

The first procedural question concerns possible ways to identify student behaviors that may be exemplars of the NCTM process standards. Rather than attempt to develop operational definitions for each of these five standards and implement a coding scheme, we decided to use a more holistic approach. Many have argued, convincingly (e.g., Schoenfeld, 1992), that the term problem solving is ill defined, due the multiple meanings ascribed to it in the research literature. Further, behaviors that demonstrate the attainment of these standards overlap considerably. For instance, when an individual solves a mathematical problem, does she not utilize sufficiently powerful representations of the problem situation and sound reasoning? If she were not able to communicate her solution to others, how would they know that she had successfully solved the problem? Do not expert problem solvers often make connections between areas of mathematics as well as connections between mathematics and its applications? As a result of these difficulties, we decided to report instead on episodes indicating mathematical thinking that include any of these process standards.

We independently examined the transcripts of the small group conferences and the projects themselves. Then, using an iterative induction process (Denzin & Lincoln, 1994), we negotiated individual inferences we had drawn regarding behaviors of the individuals in the groups. During these meetings, we re-read relevant transcript sections and presented justifications for our inferences. Only those inferences ultimately accepted by both of us were included in this paper.

Results

Here we report the findings with regard to the development of mathematical processes and the quality of the collaboration in each of the two selected extreme groups. Findings demonstrate a rich mosaic of conceptual episodes, ranging from misconceptions to "eureka" experiences.

Group A (It was the best of times.)

Under good conditions, online windows reveal deep mathematical thinking, effective collaboration, and positive social interaction. We found evidence supporting the claims that members of this group demonstrated mathematical processes and effective collaboration. Transcript excerpts are presented chronologically to give the reader a sense of the development over time.

Group A consisted of three students, two females, referred to here as Students N and D, and one male, referred to as Student B. Students N and B were visual arts majors and student B was majoring in Psychology. They were all senior-level students.

For this project, adapted from Bassarear (2001), students were to examine areas around the borders and in the interiors of pools of different sizes. In the first case, the pools are square (See Figure 1), and in the second they are rectangular with dimensions n by n+1 (See Figure 2). In both cases, students were asked to determine the number of squares around the border and in the interior of the nth figure, and the fraction of the total area contained in the interior. They were also asked to produce graphs and use them to describe the relationships they observed. The primary purpose of this project was to provide students with the opportunity to explore patterns and to express mathematical relationships verbally, numerically, algebraically, and graphically.

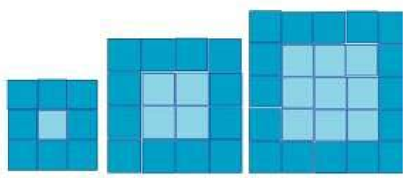

Figure 1. Square pool

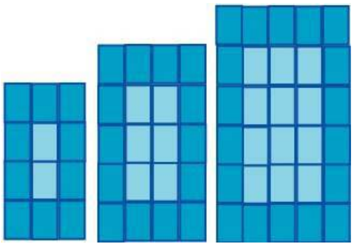

Figure 2. Rectangular pool

Initial Teamwork

The project was assigned on October 12, 2004, with the draft due in approximately one and one-half weeks and the final version due in approximately three weeks. The following excerpts contain glimpses into the nature of this group's collaboration, beginning with one student getting the process moving by offering help to the others.

> From: Student B Date: Thursday, October 14, 2004 10:51 PM
> ". . . I will be available on AOL Instant Messenger at that time, so if you need ANYTHING I am only a short click away . . . . So good luck, and please let me know if there's anything else I can do! Thanks!"

The next morning, another student reviewed the specific tasks assigned to each group member and indicated when the next posting might occur. This practice was not uncommon, as most groups in the study subdivided the workload and then shared their preliminary conclusions with the others.

> From: Student N Date: Friday, October 15, 2004 11:58 AM
> ". . . Student B will be working on problems 4+5 . . . Student D will be working on question one. . . and Student N will be looking at 3 . . . we'll also probably post some question about this project over the weekend."

Mathematical Discoveries

Later that night, Student B reported his preliminary findings. In an attached file, he presented drawings and general formulas for determining the number of squares around the border and in the interior of a square pool.

> From: Student B Date: Friday, October 15, 2004 10:52 PM
> "Ok, here's some stuff!! Take a look at this file here, it basically sums up my discoveries and graphically portrays them with explanations all in this one handy dandy file. MATHLESSON.JPG"

The next night, Student B reported a general algebraic form for determining the number of squares in the interior of the square (colored black by this group). Although he appears to have found an accurate recursive rule for determining the number of outer squares, it is unclear at this point whether he understood the geometric justification of this rule. His suggestion that the corners are increasing in size may indicate that he had not made a clear connection between the geometric change from one figure to the next and his discovery of the add-four rule. Further, Student B's posting suggests that he may have been unaware at this time that his rule for the number of inside squares is general, whereas his rule for the number of squares on the outside is recursive.

Also, the productivity of this group was impressive. Within three days, they had made major progress on the project draft. As we will see later, this stands in sharp contrast to Group B.

> From: Student B Date: Saturday, October 16, 2004 12:33 AM
> "Oh and I didn't quite specify this in the picture I don't think...so here's the makeshift formula I devised for figuring out the Nth figure: For the Inner squares: For the inner squares when working with squares, the easiest way to figure out the next number is to take the instance number of the one you'd like to solve, and multiply it by itself. Here's the formula: Black = (Instance X Instance) B = (I x I) Let's say we're solving for Instance #5: B = (I x I). B = (5 x 5). B = 25. Now you know how many inside blocks you have. . . . For Outside Squares: Going with the 5th instance as we did with the inner squares take the number of outer squares from the instance before the one you'd like, and add 4 to it. You can only increase the corners in size, so you'll always add 4 to the previous total to get the new total."

Three days later, Student D contributed her work on Problem 3, in which they were asked to calculate the fraction of the total area taken up by the inside squares for several examples, determine an algebraic rule for this fraction for the nth square, graph the fractional area versus the square size, and describe this relationship.

It appears that Student D was able to calculate the fractions for several specific squares, but she could not generalize or describe the relationship between the two variables numerically, algebraically, graphically or verbally at this point.[1] Students should be able to, "create and use representations to organize, record, and communicate mathematical ideas" (NCTM, 2000, p. 67). Yet, Student D at this point could not bring any of these four common representation modalities to bear on this problem situation. She did not develop a robust understanding of this problem situation. As a result, she was not able, for example, to consider the end behavior of this function and recognize that as the squares become infinitely large, the fraction approaches the value of one.

> From: Student D Date: Tuesday, October 19, 2004 02:35 PM
> here i added a lil more to my table adding part of 3
> a. FRACTION OF AREA IN THE POOL
> 1: 1/9 2: 4/16 = ¼ 3: 9/25 4: 16/36 = 4/9 5: 25/49
> b i have no idea about the changes i see in the fraction any ideas..?? 3c is in the table that i fixed n2 (n squared) I'm gonna try to figure out how to make a graph in the mean time Just let me know if this is ok.. thanks D

A few days later, Student N, who had been working face-to-face with Student B, made two postings regarding the task of finding the number of squares in the interiors and around the borders of the rectangular pools. Her first posting, at 2 AM, reveals that she and Student B had created an accurate formula for determining the number of squares in the interior. Despite the fact that she presented three examples along with the formula, the posting does not hint as to how they had derived it. Similarly, with her second posting, Student N reported an accurate formula without any

[1] In fact, the group never addressed this problem thoroughly. In their final project, they did not report a way to find the fraction for the nth figure, and the graph they provided was incorrect. Instead of plotting the fraction of the total area, they plotted the area inside the pool

indication of a derivation. In this case, the transcript alone did not provide sufficient evidence of the reasoning they had used.[2]

Also noteworthy in the second posting is the excitement with which Student N reported her finding. It is clear from her tone that she is enthused by her "eureka experience."

> From: Student N Date: Friday, October 22, 2004 02:01 AM
> Ok, Student B's explanation doesn't have a formula, but we sat together and figure out for 5 that to get the inner part of the rectangle, the black squares, you take the Figure # times itself, plus itself.
> (N x N)+ N (1X1) + 1= 2 (2x2) + 2= 6 (3x3) + 3= 12 etc
> Now we're working on the border....

> From: Student N Date: Friday, October 22, 2004 09:53 AM
> A-Ha! I woke up this morning with the answer! its not (N x Y) + 4, its (N x 4) + 6!!!
> i don't know how it came to me, but it LITERALLY came to me right when i woke up today . . .

The transcript excerpts from Group A tell the story of three students who collaborated well with one another as they attempted to describe patterns and relationships. The transcripts contain a rich tapestry of rudimentary understandings, possible misconceptions, and areas of difficulty.

Group B (It was the worst of times.)
Under poor circumstances, online windows reveal endless scheduling, technical difficulties, conflict, and excuses. In short, they reveal little if anything about the mathematical processes students develop. In our review of the transcripts for this group, we found numerous examples of maladaptive behavior, presented next, in chronological order.

Group B comprised three students, two females, Students A and C, and one male, Student M. Students C and M were seniors, majoring in Psychology, and Student A had earned an undergraduate degree in accounting and worked in the field for several years before returning to college to obtain her teacher certification.

Scheduling Marathon

The project was assigned on February 22, 2005. This group has approximately the same amount of time as Group A had to prepare the draft and final versions of this project. Despite the fact that the first posting occurred the next day, this group spent over five days merely trying to set up their first meeting. As the following posting reveals, only Student A attended the first meeting. As a result, with the deadline for the draft rapidly approaching, Student A decided to assign specific tasks. Clearly, this group was having difficulty collaborating effectively.

> From: Student A Date: Wednesday, March 2, 2005 1:04pm
> Since I was the only one available to work on the project yesterday, I decided just to assign out the tasks. …

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[2] Examination of this group's final project revealed that they developed both of these formulas using number pattern identification, rather than, for example, a geometric derivation.

Technical Difficulties

Shortly after midnight, on the day the draft was due, Student A found that she was unable to attach their project draft in order to submit it on WebCT. She made a total of four postings by the next morning, indicating that she had sought technical assistance from the college's computer services office. Instead of showing any evidence of this group's thinking on the project, in this case the online window provided a wealth of information on the difficulties they had been having.

Conflict

Later that afternoon, after noticing that Student A had not been in class (due to a sick child), Student C posted an announcement that she and Student M would ignore the work Student A had done but was unable to submit and would begin the project on their own.

> From: Student C Date: Thursday, March 3, 2005 2:33pm
> Student A Since you were not here today, Student M and I have decided that we were going to get a fresh new powerpoint …

Excuses

As the next two postings indicate, Group B spent a fair amount of time explaining why they were not able to accomplish various objectives. Although there were many additional postings containing various excuses, they are excluded form this report for purposes of brevity.

> From: Student C Date: Thursday, March 10, 2005 2:08pm
> I was not in class today either, due to a family emergency …

> From: Student A Date: Wednesday, March 16, 2005 10:18pm
> Student M. I can't find your phone number. Please call my cell … We can meet at the Cape May County Library in Court House. However, they do not have power point on their computers. …

Although they used the on-line environment regularly, Group B did not communicate substantively about the project. Instead of learning how these students came to develop mathematical processes, we read about the difficulties of attaching documents and the lack of available computer software. In fact, the most compelling argument for the lack of evidence regarding mathematical processes is this: Groups A and B worked on two *differen*t projects. Yet a careful review of the transcripts for group B did not contain sufficient data to draw this conclusion. Without seeing their project, we would have had no idea about the actual problem this group was examining!

Discussion

This study had two purposes: to learn about (1) students' development of mathematical processes and (2) their collaboration in a technology-enhanced course. By examining two extreme cases through on-line windows, we were able to characterize the limiting states.

Mathematical Processes

First, what kinds of understandings did these students demonstrate? The students in Group A demonstrated accurate understandings of certain mathematical ideas, misunderstandings of some, and inadequate understandings of others. For example, Student B was able to represent the problem situation verbally, numerically, algebraically, and pictorially, but he did not appear to see connections between his picture and his formulas, or the differences between his recursive and general formulas. Furthermore, we were able to track the trajectory of student understandings over time. We captured the precise moment when Student N made her important discovery.

Second, to what extent did they use technology to demonstrate these understandings? By providing regular, substantive postings, Group A utilized technology effectively to demonstrate their understandings. Their messages provided the others in the group with a clear picture of the mathematical discoveries they had made and the problems they were still facing. Besides using on-line discussion, one student in Group A utilized graphics in an attached file to communicate his findings. This action likely helped him to communicate his ideas to the others in his group.

In contrast, we learned very little about the mathematical understandings of students in Group B. Moreover, they utilized the on-line discussion tool to communicate procedural issues and problems, instead of their understandings of mathematical processes.

On-line Collaboration

First, how did these students collaborate? We found that students in Group A functioned as a community of learners. They supported one another and shared ideas and difficulties freely as they co-participated toward a common goal (Roth, 1998). Group B did not appear to collaborate. Instead of establishing joint goals and making decisions together (Hennessy & Murphy, 1999), they seemed to work as individuals or as factions.

Second, how did they utilize different communication channels? Group A used a combination of on-line and face-to-face communication, but when they met off line, they were sure to provide on-line summaries. Students in Group B also used other means of communication, but they did not report on the substance of off-line discussions.

In summary, because Group A provided evidence in their postings, we were able to draw reasonable inferences about individual (mis)understandings and group collaboration. On the other hand, for all we know, one of the students in Group B did the lion's share of the work, and the others had not developed sufficient understandings.

Conclusion

In this case study, we described the mathematical processes and collaboration of two groups of students, as seen through an on-line window. Clearly, we wish to be conservative in drawing generalization from this study. However, what lessons might we learn about the capacity of an on-line environment to provide important information? Recall that the two groups both produced high-quality projects, despite the fact that one group collaborated much more effectively than the other did. By presenting these two extremes, we hope to emphasize two principles.

First, regarding assessment of group work, the importance of reviewing the group process as well as the final product cannot be overstated. Without having access to the transcripts, we would not have known about the significant differences between these two groups. Instructors often face the difficult question of how to evaluate individuals' learning in and contributions to group projects. Having access to an on-line environment, in addition to final products, may allow instructors to gain additional insight into individuals in the group.

Second, although on-line environments that automatically generate transcripts provide a practical means for instructors to analyze student learning and collaboration, the transcripts are only as useful as the postings they contain. Why, for example, did we learn much less about the students in Group B than those in Group A? Is it because the instructor did not provide specific instructions or rules for on-line communication? Although this reason is plausible, establishing participation rules does not guarantee the development of an on-line community. According to Tu and Corry (2003), "If learners do not see the value of collaborative learning, they will focus only on achievement and will not engage effectively in collaborative activities (p.57)." Alternatively, is the difference related to characteristics of the individuals in the two groups? These questions merit further study.

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