Designing and Mathematics Learning Through the Use of Graphs

- The effects of mathematics education in design-making activities using Grafeq Program-

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Abstract: The purpose of this research is to investigate the effects of mathematics education in classroom activities using technology. The technology that was used in this study is Grafeq program. A design-making activity using graphs on a computer program was conducted with freshmen who entered the department of mathematics education (future mathematics teachers). The activity of making mathematics design was conducted over a total of three years with freshmen who entered the college in 2002-2004. As a result of the activity, the effect of improvement in students' cognitive ability and affective attitude was observed. Through the analysis of the researcher's observation and students' individual records, the effects of mathematics education in design-making activities using Grafeq program were shown. The class activity process applied in this research can be utilized in regular mathematics classes of secondary schools and for the instruction of gifted students, through diversification of its levels and methods. Therefore, this paper will introduce Grafeq program, the mathematics program used in this research, and suggest an instructional process using the program, examples of such activities, and the effects of mathematics education of such activities.

1. Introduction

NCTM, noting that the importance of mathematical knowledge has increased to live in this information society, proposed a proper use of calculators and computers in the study process of mathematical problems(see [1]). Such a proposal was suggested as 'The Technology Principle' in [2] in more specific terms like the following:

> Technology affects the mathematical contents to be instructed and raises mathematics learning ability of students. Technology reconstructs mathematical environments, and school mathematics should reflect those changes in its classes(see [2] and [3]).

Korea's 7th mathematics curriculum, also, positively proposes the utilization of technology in mathematics education, recommending the use of calculators and computers(see [4]). Under this backdrop, this research set learning mathematics through the use of technology as the research purpose and conducted a class activity of 'mathematics design' using graphs with freshmen who entered a teacher's college (future mathematic teachers) as subjects.

2. Learning Mathematics by Grafeq program.

(1) Grafeq Program

This software, which is interpreted as Graph Equation., was produced by Pedagoguery Software in Canada and can be used with ease by downloading its shareware in http://www.peda.com/grafeq (for the English Version, see [8]) or http://www.mathlove.org (for the Korean Version, see [9]), a homepage of national mathematics teachers' study group in Korea. Its usage is so simple that getting accustomed to the program takes only 10-20 minutes; and Grafeq, which enables the study of graphs of equation and the domain of inequality, is a program that draws two-dimensional plane graphs when one enters an expression as he writes on the paper in the input window without entering a separate instruction word. The easy-to-purchase materials related to Grafeq is MathLove(1998), Cho Sung-yoon (2000), and Chang Hoon and Cho Sung-yoon (2001)(see [5], [6] and [7]).

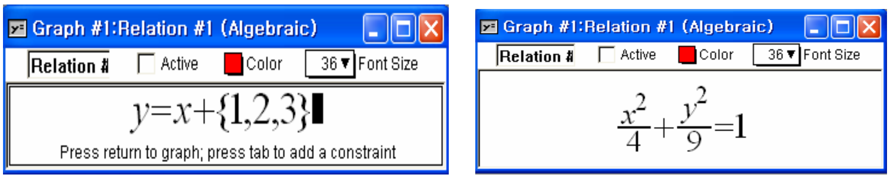


Fig. 1: The Screen to Input Equation in Grafeq

(2) Mathematics Learning in Grafeq Program.

The contents of mathematics learning, which can be practiced through Grafeq, have been analyzed in relation to the characteristics of the program.

- Understanding and utilization of various mathematical expressions:
  - Draws graphs for various numerical formula, which includes explicit function, implicit function, function with parameters, etc.
- Grasping the shape of graphs through visualization:
  - One can easily observe the changes in the graph that occur according to the change in the entered expression
- Close examination of expression and graphs:
  - This program offers various functions, including the indication of coordinate, scale line, zoom function etc.
- Understanding the meanings and utilization of the coordinate system:
  - Making it easy to understand the concepts of the coordinate system and polar coordinates.
- Understanding the concept of domain of inequality:
  - Indicating the domains of inequality in colors
- Utilization of set concepts:
  - Showing many graphs simultaneously, combining each cases like $y = x + \{1,2,3\}$.

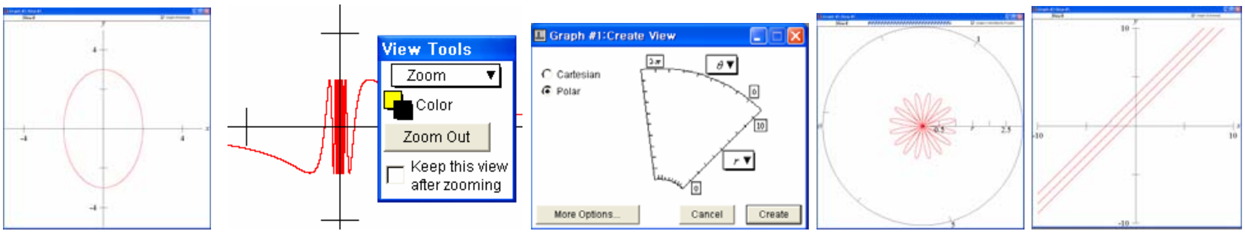


Fig. 2: Some examples of how to use Grafeq program

3. Research Design

(1) Research Subject

The freshmen, who entered the mathematics education department, teachers' college, Jeonju University(In Korea), where the researcher is working, participated in the research: 40 freshmen for each year from the years 2002 to 2004, for a total of 120 students. The reason that future mathematics teachers were chosen as research subjects was that the purpose of this research is closely related to the contents that need to be educated for mathematics teachers.

(2) Research Process

This research consists of three main phase. The first phase of this research(year 2002), the researcher conducted preliminary classroom observation. The researcher was a teacher and a observer. Throughout this time, the researcher gathered information for planning a future classroom activity. The second phase of this research(year 2003), classroom activities that were designed by a researcher on the basis of preliminary classroom observation were carried out and the results of classroom activities were investigated. Through the analysis of the researcher's observation results and students' individual records, the researcher proved the effects of mathematics education in design-making activities through the use of Grafeq The third phase of this research(year 2004), the researcher put the same classroom activities at the second phase of this research into practice again in other class and reconfirmed the effects derived from previous analysis in the second phase. This phase was for reliability of this research results.

(3) Preliminary Classroom Observation

Using the "liberal arts seminar", a compulsory class for freshman in 2002, the classroom observation was conducted over 4-week period, once a week for an hour. At the 1st class, the usage of Grafeq program were introduced to the future mathematics teachers. In the other classes, the teacher(researcher) let them use Grafeq program for learning secondary mathematics and did not provide any guide to particular mathematical activities. The researcher observed that they manipulated this program freely using secondary mathematics. But Most of them only used Grafeq program for viewing or investigating the shape of graph. From the preliminary classroom observation, the researcher concluded that specially degined mathematical activities were needed in classroom activity using program for providing a meaningful mathematical learning. The researcher attempted to plan following classroom activities in the next year.

(4) Classroom Activities

In the second phase of this research, the researcher designed classroom activities to lead participants to design various figures in the world with secondary mathematics. These activities were making design with graphs using Grafeq program. The summary of classroom activities is like the following process.

1. Introduction on the proceedings of the class
2. Simple instruction on the usage of Grafeq(Only 1st class).
3. Setting the theme of the work to be designed: Selection of one's original theme of work
4. Search for problem-solving method: Search for the proper expressions that fit the design
5. Assigning tasks: Making design with graphs
6. Task-performing activity (during class or at home)
7. Collection of task product (by group or by individual),
8. Announcement of task product-making process and the results (by individual or by group)
   : Observation of expressions used for the task,
   Question and answer session between students and the teacher
   Indication of points that need to be supplemented
9. Adjustment and supplementation of task product
   : Discussion with fellow students, Execution of joint research
10. Exhibition of produced designs

Classroom activities were conducted over 4-week period, once a week for an hour during the "liberal arts seminar" in 2003. When a new design theme was suggested at each class, the above process was repeated.

In the process of this class, the teacher(researcher) laid an emphasis on enabling the future mathematics teachers to deeply understand and apply school mathematics using technology. So the teacher participated in helping student's task performance. When the assistance was needed, the teacher provided some guidance, advice, suggestions and questions. The third phase of this research(year 2004), the researcher repeated above classroom activities with the other research subjects.

(5) Data Collection

For this research, data were collected in the following manner: The researcher observed the class continuously, recording the results of the observation. The expressions and questions of students that indicate a significant progress in terms of mathematical education during student announcement periods in each class were also recorded. The meaningful dialogue between the teacher and students during the instruction were recorded. The contents of students' announcements were all videotaped. The students were asked to write down their thoughts about the class activities(students' individual records). The researcher collected them to use as research data.

(6) Data Analysis

In order to obtain research results, the data in classroom activities after preliminary classroom observation were analyzed by three steps.

At the first step, researcher's classroom observation notes and recording materials were examined from the perspective of effects of mathematics education. In this process, we tried to find some meaningful clues supporting the effects of mathematics education in Grafeq activity. The data with these clues were classified according to result that has similar effect . These were divided into two domains. One was the effects in the affective domain, the other was the effects in cognitive domain.

At the second step, we examined students' individual records and students' announcements materials(video-taped) on the basis of the effects at each domain which was derived from the first step). From this process, we tried to confirm whether the same effects were found or not in students' individual records and students' announcements materials.

At the third step, we derived common effects confirmed from above two steps. The researcher repeated the classroom activities with other classes and investigated whether the same effects suggested in previous analysis would be present in other classes.

4. Research Results

(1) Results

Through the analysis of the researcher's observation notes, students' announcements materials and students' individual records, the effects of mathematics education in mathematical design using Grafeq were induced as follows. This effects were also confirmed in the third phase of this research.
:

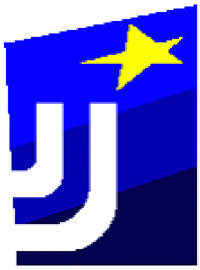
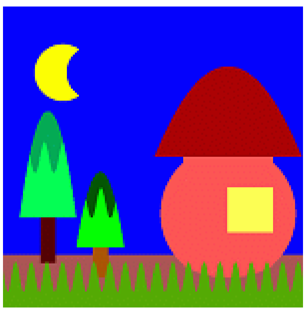
- Effects in the affective domain
  - − Change in the viewpoint concerning mathematics: acquisition of positive view of mathematics
  - − Deeper interest in mathematics
  - − Perception of one's own mathematical competence(confidence in mathematics)
  - − Experience of practicality and aesthetics of mathematics

- Effects in cognitive domain
  - Understanding and utilization of mathematical contents and knowledge
  - Experience of problem solving(trial & error strategy, A-ha experience…)
  - Integrated Mathematics Study
  - Perception of formalization and simplification

(2) Example 1: Effects in the affective domain

Following are a few examples of the applied works and some examples showing the mathematical educational effects in the affective domain, analyzed using individual records and announcements materials. But we can also see some effects in cognitive domain in the following materials.

Table 1: Some examples of student's works

Graphic Design	**Designer's Opinion** (Designer is a freshman who entered the department of mathematics education)	**The effects of Mathematics Education**
 Sign for u-turn prohibition	It was interesting to represent the signboard, which I just used to look on, using graphs. The difficult part was adjusting the sizes of circle and arrow, and their positions. In order to adjust the size of the triangle properly, the absolute value function should be well represented.	• Deeper interest in mathematics • Understanding and utilization of mathematical contents and knowledge
 Logo for Jeonju University	I didn't have any idea what kind of formula I should use. First, I tried drawing the axis of coordinates over the university trademark. Then my eyes started catching something. I think this has been an opportunity for me to gain some confidence on graphs using mathematical formulae. I think whatever I draw, I should consider it as a mathematical graph.	• A-Ha experience • Confidence in mathematics • Change in the viewpoint concerning mathematics
 The snowing village	While working with Grafeq, I realized many new aspects of mathematics that I didn't know. Drawing using mathematics… from what I have known, it was just unimaginable. So I was worried a lot, and there were trials and errors. However, I kept on drawing, and gradually I started to like what I have done and take pride in myself. I drew this picture using simple parabola, circle, and graph of trigonometric function.	• Change in the viewpoint concerning mathematics • Experience of solving problem through trial and error • Understanding and utilization of mathematical contents/ knowledge
 Calm evening scenery	This is even more meaningful because I didn't use art tools but mathematical formulas. I found out that various formulas can be put together to generate a piece of art work. I think it was a significant moment of experiencing the aesthetics of mathematics through my eyes. I wanted to express the shape of moon in a prettier way, but it took me a long time to figure out the intersection points of the two circles, which was more difficult than I thought. I like this part the most because I had the hardest time doing it. Why don't we draw various landscapes using mathematical formulas and make a calendar with them to hang in the room? Won't those people who have felt mathematics to be unfamiliar realize and fall in love with the beauty of mathematics, looking at the calendar?	• Experience of practicality and aesthetics of mathematics • Understanding and utilization of mathematical contents/ knowledge • Deeper interest in mathematics and change in the attitude on the subject

(3) Example 2: Effects in the Cognitive Domain

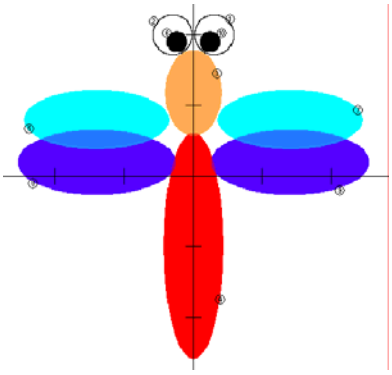
- Integrated Mathematics Study: Utilization of Various Mathematical Concepts

In class, the teacher usually asked to students:

"What expressions did you use to construct this design?".
"What mathematical concepts can you use or learn in your design?"

When asked these questions, students reviewed mathematical expressions and concepts used in his work and replied to teacher as follows:

"The expressions used to express 'dragonfly design' are circles, ellipses(In [Fig.3])"
"The mathematical concepts used to 'dragonfly design' are center of circle, radius of circle, the length of major axis and minor axis of an ellipse, parallel translation, symmetric transposition, domain of inequality(the interior and the exterior of circle and ellipse) etc."
"I think these concepts can be learned through concrete actions in the process of completing the design"



① $(x-1.5)^2+(y-10)^2=2$ ② $(x+1.5)^2+(y-10)^2=2$ ③ $\frac{x^2}{2^2}+\frac{(y-5.9)^2}{3^2}\le 1$ ④ $\frac{x^2}{0.8^2}+\frac{(y+4.9)^2}{3^2}\le 7$ ⑤ $\frac{(x+7)^2}{3^2}+\frac{(y-4)^2}{12^2}\le 3$

⑥ $(x+1.2)^2+(y-9.5)^2\le 0.5$ ⑦ $\frac{(x-7)^2}{3^2}+\frac{(y-4)^2}{12^2}\le 3$ ⑧ $\frac{(x+7)^2}{3^2}+\frac{(y-1)^2}{12^2}\le 3.6$ ⑨ $\frac{(x-7)^2}{3^2}+\frac{(y-1)^2}{12^2}\le 3.6$

⑩ $(x-1.2)^2+(y-9.5)^2\le 0.5$

Fig. 3: Dragonfly Design(Expressions used for Design)

In the analysis of dialogue between the teacher and students during class(recorded materials), The researcher concluded that constructing a design of one's own by inputting expressions on Grafeq provided the students with the opportunity to deal comprehensively with various mathematical knowledge they had gained in the middle and high school. Even more advanced from the level of learning where they simply learn and memorize numerical formulae and thus seek a solution, they experienced the activity in which they searched for necessary numerical formula on their own, expressed it properly, modified their original expressions through feedback, and thus created appropriate expressions to solve the problem. In the process of such activity, they were provided with the learning opportunity that would enhance their ability in applying mathematical knowledge and problem solving.

Many students participated in this study also said that the activity of producing design with graphs requires comprehensive employment of previously learned mathematical knowledge, including various function graphs, which were dealt with in secondary mathematics, shape and characteristics of graphs, scope of inequality, setting conditions, symmetric transposition, transformation, search for intersection point, limit of variables' scope, etc. Furthermore they expressed that this activity requires not just the recall of knowledge but application faculty, creative power and imaginative power.

- Perception of formalization and simplification

In this research process, the teacher tried to provide questions to guide students. From the analysis of classroom observation, dialogue between the teacher and students and the student's works, we found out that student's mathematical activity could be enhanced by teacher's guidance.

For example, the teacher suggested the following questions to students in the class.

"What mathematical concepts can you use to paint color in your design?"
"If you change the coefficients of expressions in your design, what kind of changes will appear in your design?"
"Can you make your design by reducing the number of expressions without any changes in shape?"
"Can you take several expressions with the same form and put them together in one expression?" etc.

By observing the process of mathematical design product which was dealt with during the class, the researcher confirmed how the expressions which were dealt with in secondary mathematics can be applied and how mathematical ideas can be given shape through the Grafeq program.

The butterfly design that is introduced in [Fig. 4] is an example of utilizing linear equations and the equation of circles any high school student can use with ease. A student who was asked by 'what mathematical concepts can you use to paint color in your design?' changed his design as follows.

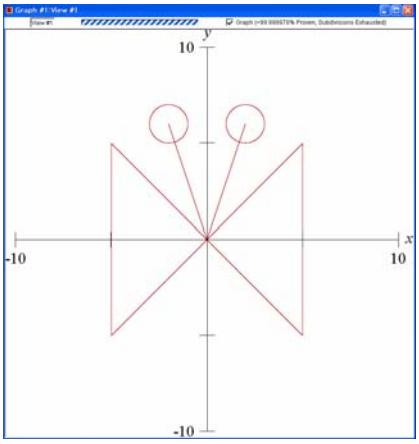


Fig. 4: Linear function and circle used

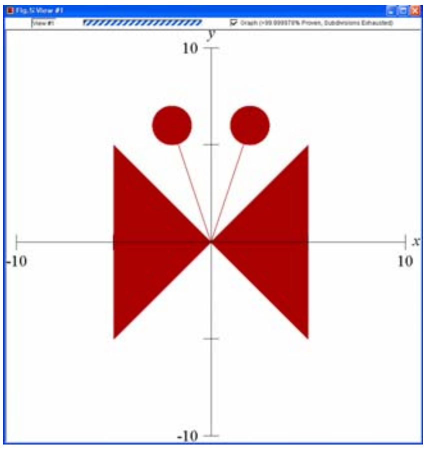


Fig. 5: Application of Inequality concept

[Fig. 4] is the expression of only the edge of the design and [Fig. 5] is the result of filling the inside of the design with the inequality concept. By observing the process of this student's design product, the researcher became know that student's use of mathematical expressions output were renewed again and again whenever the teacher suggest a question to him. When the teacher asked him, 'can you make your design by reducing the number of expressions with the same shape?', the student revised expressions used in his design and entered the new expressions.

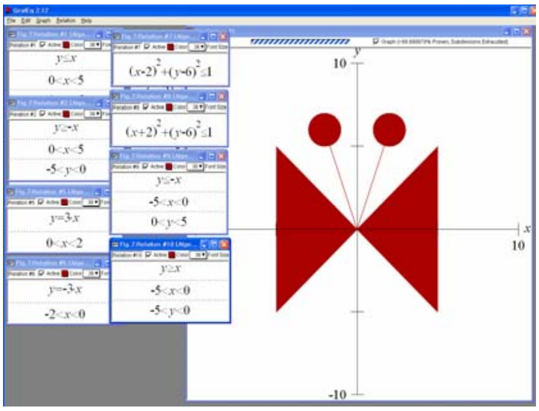


Fig. 6: Many expressions are used

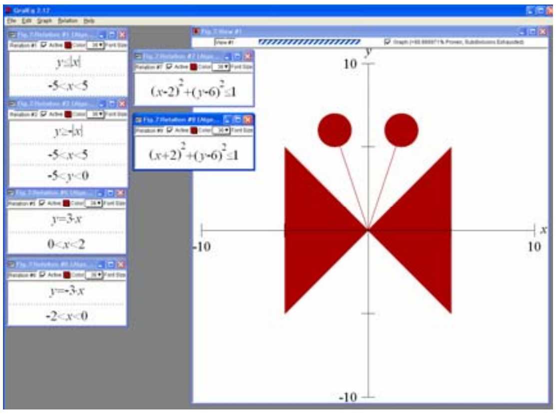


Fig. 7: Absolute value function used

The student understood that even the identical designs can be expressed by using different numerical formulae as shown in [Fig. 6] and [Fig. 7]. The number of numerical formulae and the refinement level of expression employed in a design product have a close correlation with the producer's mathematical thinking faculty. The student attempted to simplify the expressions if possible. In his activities, we observed the same design in an integrated formula like [Fig. 8], applying such mathematical knowledge as simultaneous inequality concept, graph divided by section, etc.

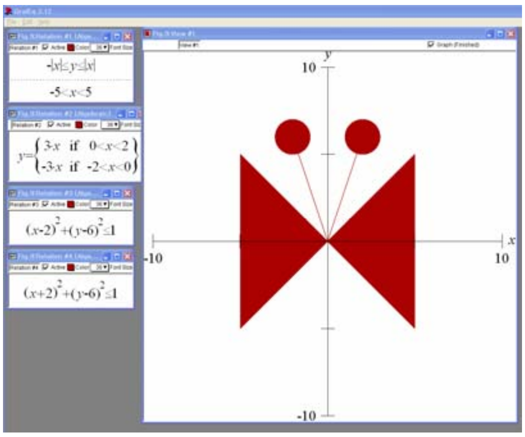


Fig. 8: Integration of expressions: More compressed expression

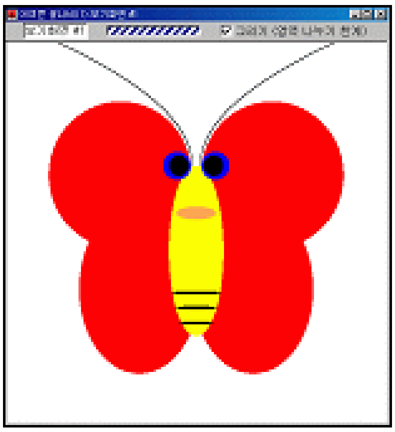


Fig. 9: More refined design

The work of mathematically integrating and compressing expressions as shown above has a close correlation with the growth in mathematical knowledge and an advance in mathematical thinking faculty.

Sometimes, the teacher let students to make more realistic design after the discussion about their design. [Fig. 9] is an example of a student's product, which realistically expressed a butterfly, using more complicated expressions.

Looking at the expressions used for [Fig. 9], one can observe the use of the equation of ellipse, irrational function, which is more advanced than using a linear equation and the equation of circle in [Fig. 6] . In addition, watching closely the coefficients employed in the expression, one can find the scope of coefficients were extended to the rational number for more refined expression, while only integers were employed as coefficients in [Fig. 6]. The level of refinement and ingeniousness of a product differs according to the producer's mathematical knowledge and thinking faculty. Therefore, even for the same design products, the expressions used can differ according to the producer's level of middle school, high school or college student.

5. Conclusion

The paper has significance as a study which attempted a new approach of learning methodology of mathematics using applied technology. Considering the effects of mathematics education obtained as a result of the learning process used in the study, mathematical design activities using graphs can be directly applied in mathematics classes at middle and high schools.

Mathematics teachers may also utilize the works of mathematical designs to aid the teaching and learning of functional graphs. They can find, in the design works, the mathematical formulas related to what they are currently teaching and use them to induce the students' interest in the introduction part of the classes. It is also possible to guide the students to mathematically analyze the formulas and conditions used in the works. For upper grade students whose mathematical knowledge is deeper and more varied, activities of drawing more refined mathematical formulas from given design works can be attempted. Lastly, it is possible to hold an exhibition of students' works in mathematics if many good works are produced.

It is hoped that various technologies, including the Grafeq program introduced in the study, will be utilized, and studies for developing teaching and learning methodologies in mathematical education will be actively put into practice in middle and high schools.

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