

# Analyzing the Performance of Grade 6 Students in Dynamic Geometry Manipulative Tasks: A Quantitative Approach

*Ka-Lok Wong*

[klwong3@hku.hk](mailto:klwong3@hku.hk)

The University of Hong Kong

*Arthur Man-Sang Lee*

[amslee@hku.hk](mailto:amslee@hku.hk)

The University of Hong Kong

*Kwok-Chun Tang*

[kctang@hkbu.edu.hk](mailto:kctang@hkbu.edu.hk)

Hong Kong Baptist University

**Abstract:** This paper reports the results of a quantitative analysis of students' performance in ten dynamic geometry manipulative tasks. More than 250 Grade 6 students from about 90 primary schools in Hong Kong attempted the tasks in a mathematics competition. These tasks, set up with the software C.a.R. but accessible with an ordinary Java-enabled browser, require students to drag points (mostly vertices of polygons) in pre-constructed geometric drawings so as to obtain specific areas or shapes. They are designed in such a way that students have to exercise their understanding in elementary geometry while continuously transforming the geometric figures on the screen. Apart from these geometric tasks, there is also a paper (in multiple-choice format) that calls for general mathematical reasoning and knowledge. Correlation between the scores on the multiple-choice test and the performance on the dynamic geometry tasks is also studied. The correlation is only moderately positive, which tends to suggest that the dynamic geometry tasks are probing a kind of ability different from the general mathematical competence expected of a Grade 6 student. Results of the students in the ten dynamic geometry tasks are also analyzed in an exploratory manner. Factor Analysis renders an extraction of four factors, three of which can be possibly interpreted with partial reference to the content knowledge involved, namely, knowledge about symmetry, area and congruence. But, with a careful examination of the geometric contents as well as the design of some of the tasks, we suggest a distinctive element of exploratory interaction, as opposed to working with formulas and computation, that should help characterize the factors at a deeper level.

## 1. Introduction

Dynamic geometry tools for mathematics education are becoming more and more accessible and friendly. In terms of technological feasibility, one may envisage its incorporation into the learning, teaching and assessment of geometry in a wide range of settings. Dynamic geometry content can be easily created with subject-specific software, such as Geometer's Sketchpad or Cabri-Géométrie, or the more general authoring tools such as Flash or Java. These tools can help produce in a dynamic geometry (hereafter shortened as DG) environment interactive learning units, tasks for challenging students or testing their basic geometric knowledge. (Mathsnet, Project Interactivate and National Library of Virtual Manipulatives are examples easily accessible in the Internet.) Students can work on focused tasks with pre-constructed figures or engage in open exploration with high degree of control. However, no matter what kind of activities are involved, merely acting on DG objects and interpreting their behaviors may already require skills and knowledge different from those required for classical geometry in the traditional mode of content delivery (Whiteley, 2000).

There is thus the need to better understand the nature of the geometric skills and knowledge pertaining to the new DG environment. This is particularly so when we are considering relevant changes in assessment practice, since advances in technology have granted various options of conducting assessments in the electronic platform. No matter whether they are done by minimal or radical modification of traditional paper-based tests, we expect assessment tasks to have their designs and contents fitting the purpose and context. What geometric skills and knowledge do we want our students to learn? What can we tell from the performance of students in certain kinds of geometric tasks? Once we consider these questions in context (i.e. in the DG environment given the present curriculum), we cannot but come to notice the possible shift in knowledge being tested. In this paper, we report our analysis of students' performance in ten DG manipulative tasks. Hopefully, it would help us understand better what we can tell from the performance of students in certain geometric tasks, which may in turn cast light on the implication of incorporating DG technology in the design of assessment tasks for geometry at elementary level.

## **2. Method of Study**

### **2.1 The Context**

Although the title of this paper seems to suggest a statistical survey, we must at the beginning clarify the context out of which the data have been generated. To put it shortly, we do not claim any representative sample out of a rigorously designed statistical survey. And the following subsections will thus appear to deviate from the usual descriptions of subjects, instruments, procedures and so forth. Instead of the usual path from well-formulated research questions or hypotheses to research design then to data collection, it is in the context of some ongoing events that our understanding of students' performance in DG environment gradually emerges with the aid of statistical analysis.

In early 2005, a mathematics competition was organized for Primary 6 (i.e. Grade 6) students in four separate districts in Hong Kong. Each primary school in the districts was invited to send some students to participate. Without detailing the logistics of the competition, suffice to know that more than 250 students from about 90 primary schools in those four districts attempted a set of ten DG manipulative tasks, which was one component of the competition. Whereas the tasks set in the DG environment were believed to be novel to most if not all participants, the geometric knowledge needed was supposed to be covered in their current mathematics syllabus. It calls for basic understanding of size of angles, area of triangles, congruence of polygons and symmetry of shapes or patterns. More details of these DG manipulative tasks will be given in Section 2.2. It should be noted yet that prior experience with or substantial knowledge about DG was not assumed on the part of the students. Notwithstanding the fact that we did not start with a statistically representative sample of the general population of Grade 6 students, the reasonably large number of students from many schools, each being a contestant in an inter-school mathematics competition, justifiably constitute a good sample of the mathematically high achievers at Grade 6 in the four districts concerned.

In the same competition, these students also completed a multiple-choice test of 30 items designed to assess their general mathematical competence. Whereas a few items were designed to test students' logical reasoning and general analytic ability, most of the test items were largely connected with the content knowledge specified by the primary mathematics syllabus. However, it could be argued that all the test items required non-routine application of such elementary mathematics as counting, enumeration, arithmetic operations, measuring with areas and volumes,

fractions and simple proportions. The total score on this multiple-choice test thus grants us a rough measure of the general mathematical competence of the Grade 6 participants.

Both the multiple-choice test and the test with DG manipulative tasks, conducted separately, lasted for 45 minutes. As observed, most students completed the questions in good time. In other words, time constraint did not appear to have significant impact on their performance.

## 2.2 Designing the DG Manipulative Tasks

In the design of this set of tasks for Grade 6 students who are coming to finish their primary studies, the major considerations are the geometric knowledge that they may have at this level and the capabilities provided by the dynamic geometry tool. In particular, in the DG environment, students can continuously transform a shape under designated constraints (which are invisible to the students themselves). This manipulation is actually an interaction with the geometric figure because the student, when acting on the figure, is supposed to notice and thus respond to the changes in various geometric properties such as area, length, angle size and symmetry, either intuitively or based on the given measurements (including the use of square grids). In our case, the manipulative tasks were created with the DG software C.a.R. and were presented as Java applets embedded in web pages. Students accessed the pre-constructed dynamic figures through Java-enabled web-browsers. They needed not have any experience in using DG software. To finish a task, they were only required to drag a certain movable point or more (with the other points fixed) to modify the figure until some particular conditions were satisfied. For example, the first task in the test, shown in Figure 1(a), asks a student to drag the only movable vertex F so that the two triangles have equal area. To satisfy this requirement, he/she can put the point F anywhere 4 units from the base DE.

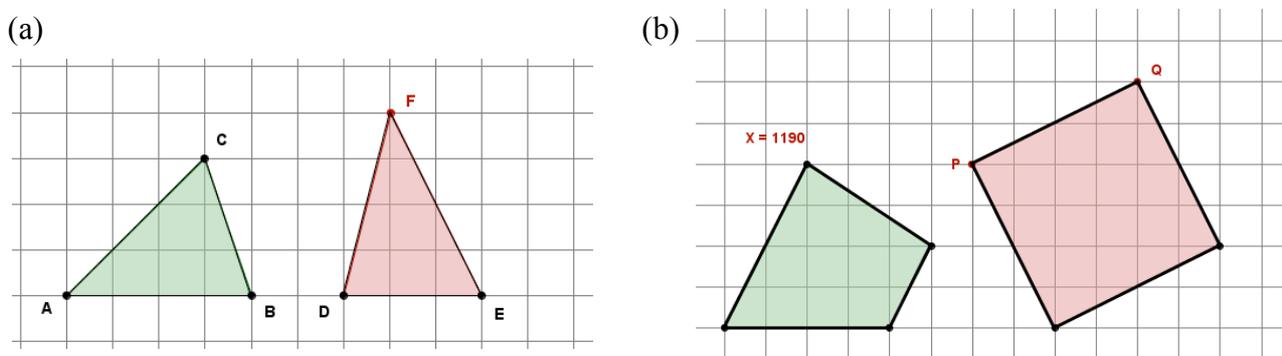


Figure 1 Examples of dynamic geometry manipulative task: (a) Drag F so that the triangles have the same area. (b) Drag the points P and Q (movable only on a horizontal and a vertical line respectively) so that the quadrilaterals are equal in shape and size.

Each of these manipulative tasks has a parameter that encodes a certain geometric property of the figure in relation to the required conditions. In the previous example, according to the position of F, the difference between the areas of the triangles is computed and designated as the parameter. Instead of the whole geometric figure that a student thinks to have attained for the solution, this parameter is the only thing to be submitted. Noteworthy is that such a parameter, encrypted with the aid of a function devised by the designer, presents itself to the student in disguise of a

strange-looking value which does not tell anything to the student when he/she is tackling the task. This is the essential feature that leads to a quantitative approach to students' performance in these geometric tasks because the value of the parameter gives, so to speak, a numerical measure of a geometric figure as to whether, and in a certain sense, how well the required condition is satisfied. A student's performance on a task is considered successful when he/she can attain the required figure with reasonable precision which is to be gauged by the value of the parameter. Figure 1(b) provides another example together with its corresponding parameter shown as a value of  $X$ .

Based on the reported value of  $X$  in a DG manipulative task, a score will be given, according to how far the result deviates from the correct result. The maximum score for each task can be 1, 2 or 3, depending on its complexity. The full score on these ten DG manipulative tasks is 24.

### **2.3 Seeking to Understand Students' Performance in DG Manipulative Tasks**

We seek to understand the performance of these Grade 6 students in the DG manipulative tasks. However, this question of performance should not be a simple one in terms of a high or low score, which has already been answered by way of identifying the winners in the mathematics competition. This is unavoidably the case when a student receives a single number as his/her total score on the ten manipulative tasks. It is however in a qualitative sense that we want to understand the students' performance given that we are working with a set of quantitative data. While this kind of DG manipulative tasks is still developing along with our fledgling knowledge about quantifying students' performance in a novel environment, we attempt to answer the following questions by a quantitative analysis of the data:

- (a) How well do the students (a group of high achievers in our case) perform in this kind of DG manipulative tasks?
- (b) Can we reduce the students' performance in the DG manipulative tasks (now represented by ten scores) to a few factors that would help characterizing their performance?
- (c) Is there any association between students' performance in this kind of tasks and their general mathematical competence?

Simple descriptive measures will give us an idea of the general performance. But, as explained above, the test items were designed according to our understanding of how a Grade 6 student should be able to do in the area of geometry. Structure in the ten items themselves, if any, might be due largely to the mathematical topics and contents that usually govern our conceptions when we are setting questions for the students. It may lack a strong reference to students' actual performance in a variety of questions in various contexts. This is especially true when we are now talking about problems in the new environment of dynamic geometry. We will thus employ factor analytic techniques and correlational analysis to explore possible answers to questions (b) and (c).

## **3. Results and Analysis**

### **3.1 General Findings**

More than 250 Grade 6 students attempted the ten DG manipulative tasks during the mathematics competition. They came from about 90 primary schools in four widely separated districts in Hong Kong. Due to missing data which defies factor analysis, we will employ data from 229 students for the factor analysis of the resulting scores on these ten manipulative tasks (see Section 3.2 below).

However, due to an unexpected problem with the scores on the multiple-choice test in one district, only 185 students will be taken into account for the general analysis involving data other than those from the manipulative tasks, especially when we concern ourselves with the association between students' performance in the DG manipulative tasks and their general mathematical competence.

The performance of the 185 students as a whole on the DG manipulative items may be considered as moderately satisfactory. Of the full score of 24 points, the students on average score a little above half the total ( $M = 13.1$ ). Their performance varies considerably, with the scores taking up the full range from 0 to 24 ( $SD = 5.1$ ). The distribution of scores skews only slightly to the left.

For the 30-item multiple-choice test, the score was worked out with various weightings given to individual items. This was done by the organizers of the competition in order to recognize the varying significance and difficulty among the items. As a result, the full score on this multiple-choice test is set to be 100. The average score is 52.3 ( $n = 185$ ,  $SD = 15.3$ , range = 15 – 90). The scores of these 185 students distribute quite symmetrically about the mean.

Relation between the students' performance in DG manipulative tasks and their general mathematical competence as reflected by the scores on the multiple-choice test is examined by an analysis using Pearson's correlation coefficient. An analysis of the data from the 185 students reveals a moderately positive correlation,  $r(183) = .419$ ,  $p < .0005$ .

### 3.2 Factor Analysis

For a better factor analysis of the responses to the DG manipulative tasks, we try to have the largest sub-dataset. As this factor analysis concerns only with the students' performance in the DG tasks, the scores on the multiple-choice test are irrelevant. Thus, we have gone beyond the restrictive dataset of 185 students reported in the above general findings. But since factor analysis works only if each of the students involved has produced responses to *all* the ten tasks, we exclude those who have failed in this respect. As a result, among all the participants, we take the data from 229 students in the factor analysis to follow. With the students' scores on the ten DG tasks (Tasks 1 through 10), we conduct a principle component analysis with varimax rotation method ( $N = 229$ ). Four principal components (i.e. *factors*) are extracted with eigenvalues greater than 1.0 (Kaiser's criterion) and explain a cumulative 56.00% of the total variance. The percentages of variance that these orthogonally rotated factors, Factors I to IV, account for are 16.72%, 14.62%, 13.77% and 10.89% respectively. Table 1 overleaf shows the measures of the association between each of the ten tasks and each of these four orthogonally rotated principal components.

Interpretation of the meaning of these factors will be discussed in Section 4. Here we assume the four factors and try out further analysis, particularly that of scores based on Factors I to IV, to explore their relationships with the other variables available to us.

### 3.3 Factor-Item Percentage Score

Since each factor is largely accounted for by a certain number of tasks, we convert a student's sub-total score on the tasks correlated with each factor as a percentage (namely, a number between 0 and 1). We call this *factor-item percentage score*, which, in a certain sense, gives the score of the student on a particular factor. For example, Factor I correlates more with Tasks 6, 7 and 10 which

Table 1 *Loadings on the Four Orthogonally Rotated Principal Components*

	Components			
	Factor I	Factor II	Factor III	Factor IV
Task 7	.805			
Task 6	.790			
Task 10	.445			
Task 1		.705		
Task 2		.670		
Task 8		.637		
Task 3			.703	
Task 4			.608	
Task 5			.581	
Task 9				.927

Extraction method: Principal Component Analysis.

Rotation method: Varimax with Kaiser Normalization.

Note:  $N = 229$ ; only factor loadings over .40 are reported.

<sup>a</sup> Rotation converged in 6 iterations.

have their respective maximum scores 2, 3 and 2 (totaling to 7); therefore, if a student gets 1, 3 and 0 on Tasks 6, 7 and 10 respectively, he will get 4/7 (i.e. 0.57) as his factor-item percentage score for Factor I. As a result, for each student, we reduce his/her ten scores on the DG tasks to four factor-item percentage scores (hereafter abbreviated as Score I, Score II, Score III, and Score IV).

In order to involve the scores on the multiple-choice test in further analysis of these scores based on the factors, we work with the smaller sub-sample of 185 students (as explained above in Section 3.1). Table 2 gives a summary of Scores I to IV, which shows their means lying roughly in the middle and considerable variations for all but Score IV. The last one behaves strangely because, as shown in Table 1 above, Factor IV is constituted only by Task 9 on which more than two-thirds of the students receive a zero score.

Table 2 *Summary Statistics of Scores I to IV*

	Score I	Score II	Score III	Score IV
Mean	0.52	0.58	0.61	0.28
Standard Deviation	0.34	0.25	0.34	0.44

To examine the relationships among the scores on different aspects, we have Table 3 which gives the inter-correlations (Pearson's correlation coefficients) between the score on the multiple-choice test, the total score on the DG manipulative tasks, and the four factor-item percentage scores. The weak correlations among the four scores themselves are expected, because the factors are resulted from orthogonal rotation. And the moderately positive correlation between the total score on the DG manipulative tasks and the score on the multiple-choice test has been reported above (Section 3.1). Now, we can also see that all the four Scores I to IV consistently show a weak correlation with the total score on the multiple-choice test ( $r(183)$  are all less than .30 and significant with  $p < .01$ ).

Table 3

*Covariance Matrix for Scores on the Multiple-Choice Test and DG Items, and Scores I to IV*

	Multiple-choice test	Score on DG tasks	Score I	Score II	Score III	Score IV
Multiple-choice test	—	.419**	.268**	.292**	.266**	.275**
Score on DG tasks		—	.685**	.553**	.767**	.411**
Score I			—	.119	.253**	.208**
Score II				—	.234**	.121
Score III					—	.178*
Score IV						—

Note:  $n = 185$ .

\*\* Correlation is significant at 0.01 level (2 tailed)

\* Correlation is significant at 0.05 level (2 tailed).

On the other hand, all the Scores I to IV are positively correlated with the total score on the DG manipulative items, with varying strengths – from moderately positive ( $r(183) = .411$ ) to strongly positive ( $r(183) = .767$ ), all statistically significant with  $p < .01$ . But it is interesting to note that both Scores I and III show a stronger positive correlation than Scores II and IV do:  $r(183) = .685$  and  $.767$  respectively for Scores I and III, as compared with  $r(183) = .553$  and  $.411$  respectively for Scores II and IV. In other words, the general performance in the DG manipulative tasks correlates more strongly with Factor III (i.e. Tasks 3, 4, and 5) and Factor I (i.e. Tasks 6, 7 and 10) than with the other two factors.

## 4. Discussion

### 4.1 Taking a Vantage Point

Judging from the average performance of the students as a whole, it is hard to say that a mean score barely above 13 (out of 24) indicates a very satisfactory result in those dynamic geometry tasks, especially for the present case wherein the students are generally considered as high achievers in Grade 6 mathematics. Whereas one could argue that the DG tasks might be just too novel for the students to exercise their current knowledge, one should also notice a similar level of performance (only a moderate value of average score) in the multiple-choice test. To put it more positively, the distributions of the scores on both the DG manipulative tasks and the multiple-choice test, together with their respective means and standard deviations as reported in Section 3.1, show that the tasks in general have been set at a reasonable level of difficulty and differentiate the students reasonably well across a wide range of scores. This is no doubt a desirable condition for further analysis.

At the same time, we observe a moderately positive correlation between the performance in the DG tasks and the score on the multiple-choice test. Recall that, as mentioned above, the multiple-choice test, set as non-routine problems albeit in a traditional paper-and-pencil mode, is largely connected with what the students have learnt in such elementary mathematics as counting, enumeration, arithmetic operations, measuring with areas and volumes, fractions and simple proportions. This correlation of only moderate strength seems to suggest that the DG manipulative tasks are probing a kind of ability considerably different from the general mathematical competence recognized in the

traditional curriculum. Obviously, this suggestion aligns itself with the nowadays common understanding about the nature of technology-based mathematics teaching and learning. In their final analysis of what digital technologies (such as DG software) can take from and bring to research in mathematics education, Hoyles and Noss (2003, p.341) assert that “DG-maths is not the same as maths *per se*, and that – by implication – neither is the knowledge that learners develop.” The problem is: how different is it? Can we characterize it in one way or another? While many researchers have contributed much by giving qualitative accounts in great detail of students’ learning and/or teachers’ teaching in the DG environment, the quantitative findings reported above seem to have shown us another direction.

#### 4.2 Characterizing Students’ Performance in the DG Manipulative Tasks

One notable finding out of the factor analysis above is that four factors have been extracted from the ten dynamic geometry tasks. Before we can interpret the factors, we should consider what are involved in the DG tasks. We have already provided two examples in Section 2.2 (which are actually Tasks 1 and 5 respectively). Within limited space of this paper, we cannot show all the tasks with that level of details, but only provide in Table 4 a brief description of *what* the students are required to do in each of the tasks, grouped according to the factors to be interpreted.

Table 4 *Description of the DG Manipulative Tasks in Different Factor Groups*

Factor I	Task 7	Make a hexagon with two oblique lines of symmetry. Drag one vertex.
	Task 6	Make a hexagon with two lines of symmetry. Drag two vertices.
	Task 10	Make a regular hexagon by intersections of six equal circles.
Factor II	Task 1	Make a triangle with area equal to another. Drag one vertex.
	Task 2	Divide a quadrilateral into four regions in two pairs with equal sums of areas.
	Task 8	Divide a sector into three regions with areas in a given proportion.
Factor III	Task 3	Make a quadrilateral (and in turn a triangle) with a specific area.
	Task 4	Make a quadrilateral congruent to another. Drag one vertex.
	Task 5	Make a quadrilateral congruent to another. Drag two vertices.
Factor IV	Task 9	Adjust a square inscribed in another so that its area takes up a certain fraction inside.

It seems reasonable to consider a task in terms of the content knowledge. In this case with geometric tasks, we tend to organize them according to the geometric knowledge that is involved. For example, a glance of the tasks listed above in Table 4 may lead to an interpretation of the factors in the following manner or alike:

- Factor I: symmetry of (two-dimensional) shapes
- Factor II: area of simple geometric shapes
- Factor III: shapes that are equal in areas and shapes (congruence)
- Factor IV: area (or is it fraction?)

However, once we have started this interpretation, we should come to two points of observation. Firstly, the basic idea of factor analysis is to classify into somewhat independent factors those variables which are correlated. So, Factor IV (i.e. Task 9 only), though largely independent of the other factors, should not be of any particular interest insofar as it does not correlate with any other variables. Without any other task(s) for comparison within the same factor, we cannot identify

whether it concerns more with area or fraction or else. (Thus, much less weight will be given to Factor IV in subsequent discussion.) Secondly, a careful perusal of the contents should reveal the inadequacy of such content-based interpretation of the factors. Although basic understanding of the area of triangle (in particular, area in relation to the base and the corresponding altitude of a triangle) seems to play a central part in both Tasks 1 and 3, they are statistically associated with different factors. There must be something other than the simple knowledge about the area of triangle that puts these tasks under different factors.

In Task 1, as shown in Figure 1(a), two triangles ABC and DEF are given, the former being fixed and the latter changeable. Students are asked to drag the only movable vertex F so that both triangles become equal in area. To arrive at such a triangle, a student can put the vertex F anywhere 4 units from the base DE. On the other hand, Task 3 (see Figure 2) also calls for the area of triangle ABC; Yet

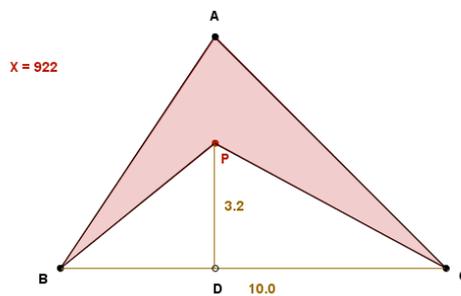


Figure 2 Dynamic geometry Task 3: P is on the height AD of triangle ABC. PD is adjustable and  $BC = 10$ . Drag P so that the shaded (red) area is 18.

with only the base BC given but the corresponding height unknown, the problem situation creates a need for the student to interact with the geometric drawing. That is to say, although students have to make use of their knowledge about area of triangle in both Tasks 1 and 3, they may react to the dynamic figures in different ways. In Task 1, without any idea of putting the vertex F at a specific height, trial-and-error type of dragging on the point may not be very fruitful. Thus, the area formula, rather than the dynamic figure itself, should be crucial to the solution. In contrast, in Task 3, notwithstanding good knowledge of the area formula for a triangle, a student without any attempt of explorative dragging P along AD would get stuck with the apparently missing information and the shaded area would appear unknown throughout. Unlike Task 1, students on Task 3 cannot just *read* the diagram and *calculate* the answer by simply knowing the area of triangle *as a formula*.

The above comparison of Tasks 1 and 3 casts light on our interpretation, granting us a perspective upon something beyond content knowledge. Just as interactions or exploratory actions on the diagram seem to play a crucial role in tackling Task 3, the same kind of exploratory skill is important to successful performance in Tasks 4 and 5. Exploration by dragging a movable point is in itself a means to better understand how the geometric shape varies, and, through such an interaction with the dynamic figure, comprehension of the lengths and angles involved and thus a correct comparison (for congruence) of shapes is achieved. To our interpretation, this partly explains why Tasks 3, 4 and 5 are associated with each other to constitute Factor III. Nevertheless, the same demand for exploratory skill is noticeable in other tasks such as Tasks 6, 7 and 10. But while symmetry is an obvious characteristic of the tasks under Factor I, we tend to believe that

exploration in this case is geared towards possible variation of the figure in search of a match with a visual image of symmetry, which is arguably more demanding than the comparison of more tangible lengths and angles. This could explain, at least partly, why Score I is less than Score III.

As opposed to tasks associated with Factors I and III, tasks under Factor II (such as Task 1 detailed above) may rely more on the ability or knowledge (e.g. applying the area formula) to work out the numerical measure in need, rather than on the visual clues emerging from the interaction with the dynamic figure. That is, dragging on and interacting with the DG figure is not necessarily the crucial strategy. Moreover, noteworthy is that this kind of reliance on formula works and computation resembles what students (at least those in Hong Kong) usually get themselves trained up in the traditional curriculum. It is thus not surprising to have the above statistical finding that the general performance in the DG manipulative tasks correlates more strongly with Factor III and Factor I than with Factor II. When pursuing further how well each of the factors determines the general performance in the overall DG manipulative tasks, we try dividing the 185 students into two subgroups, higher-DG-score ( $> 13$ , and  $n = 98$ ) and lower-DG-score ( $\leq 13$ , and  $n = 87$ ). An independent-samples  $t$ -test with each factor, without assuming equal variances, shows a significant difference in the factor-item percentage score between the two subgroups (for Factors I to III,  $t(179.6) = 9.22$ ,  $t(152.7) = 6.22$  and  $t(158.1) = 11.71$  respectively, all with  $p < .0005$ ). The difference in Score III (difference = 0.45) and Score I (difference = 0.38) are much more substantial than in Scores II (difference = 0.22). Meanwhile, it is very interesting to observe almost the reverse case (albeit less prominent due to the weaker correlation between the performance in DG tasks and that in multiple-choice test) when we conduct a similar independent-samples  $t$ -test for the effects of the factors on students with better and weaker performance in the multiple-choice test. With two subgroups, higher-MC-score ( $> 50$ , and  $n = 93$ ) and lower-MC-score ( $\leq 50$ , and  $n = 92$ ), the difference is most significant for Factor II ( $t(181.1) = 3.76$ ,  $p < .0005$ ) whereas it is less significant for Factor I ( $t(182.4) = 3.43$ ,  $p < .005$ ) and Factor III ( $t(182.9) = 2.54$ ,  $p < .05$ ).

In short, Factor I and III, both of which revolve around an ability (a strategy, or a willingness?) of exploratory interaction, seem to be crucial factors determining the performance in dynamic geometry. In contrast, Factor II, though a constituent part in the DG performance, is connected more with traditional kind of knowledge based on formula works and computation. The above results have also alerted us to the need for an improvement of students' capability to explore and visualize geometric properties rather than mere computation with formulas. It is hoped that the above analysis and interpretation will help us understand better what constitutes the success of students in their performance in the dynamic geometry manipulative tasks. This would in turn support further integration of computer technology into the mathematics curriculum.

## References

- Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick & F.K.S. Leung (Eds.), *Second international handbook of mathematics education: Part one* (pp.323–349). Dordrecht: Kluwer Academic Publishers.
- Whiteley, W. (2000). Dynamic geometry programs and the practice of geometry. Paper distributed at the Ninth International Congress on Mathematical Education (ICME 9), 31 July – 7 August, Tokyo. (Available at <http://www.math.yorku.ca/Who/Faculty/Whiteley/Dynamic.pdf>.)