

Using a Graphing Calculator to Explore Pre-university Level Mathematics – Some Examples Given in an In-service Course

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Abstract

Graphing calculators without a built-in computer algebra system will be permitted for use in all advanced level (pre-university level) national mathematics examinations in Singapore from the year 2006. Teachers in Singapore who are teaching mathematics at the advanced level are expected to be proficient users of a graphing calculator so as to teach effectively with a graphing calculator and prepare their students adequately for the incorporation of graphing calculators in assessment. To address the training needs of the teachers, the author had conducted, in the first half of the year 2005, three classes of an in-service course on the use of the TI-84 Plus graphing calculator in the teaching and learning of advanced level mathematics. This paper discusses briefly the role which graphing calculators could play in Singapore's new advanced level mathematics curriculum, presents some of the examples given in the course and reports on the participants' evaluation of the course.

Introduction

The use of graphing calculators in the teaching and learning of mathematics has received increased attention in recent years. Proponents of graphing calculators generally contend that using such hand-held technology in the classroom will save time on tedious calculations and remaining time can be used for investigation of different aspects of a given problem (Drijvers and Doorman, 1996). It is also argued that the availability of such hand-held technology enables students to access numerical, graphical and symbolic representations of a given problem that will help the students find connections among these representations to make sense of the problem (Adams, 1997; Demana, Schoen and Waits, 1993; Dick, 1992; Hennessy, 1999). Indeed, research has found that graphing calculators, if exploited effectively, can create an environment in which students actively construct their knowledge by exploring realistic situations and stimulate the exploratory and dynamic approach to mathematics, a more positive view of mathematics, and a more flexible behaviour in problem solving (Dunham and Dick, 1994).

Recognising the graphing calculator as an important tool in teaching and learning mathematics, the Ministry of Education of Singapore has allowed, since the year 2002, the use of graphing calculators without a built-in computer algebra system (CAS), also known as non-CAS graphing calculators, in the advanced level (pre-university level) national examinations for Further Mathematics, a subject taken by students with a keen interest in, and have an aptitude for, mathematics. A revised advanced level syllabus and curriculum will be introduced in the year 2006 and a significant change in the mathematics curriculum is the incorporation of non-CAS graphing calculators in assessment in all advanced level mathematics subjects.

Following such a change in the curriculum, all teachers who are teaching mathematics at the advanced level are expected to be proficient users of a graphing calculator so as to teach effectively with graphing calculators and prepare their students adequately for the incorporation of such technology in assessment. To address the training needs of these teachers, the author of this paper had conducted three classes of an in-service course in the first half of the year 2005 on the use of the TI-84 Plus graphing calculator (hereafter TI-84+ for brevity) in the teaching and learning of advanced level mathematics. In what follows, the role which graphing calculators could play in the

new advanced level mathematics curriculum will be discussed, and details of the in-service course will be provided, before some examples given in the course are presented. Results of participants' evaluation of the course will also be reported.

The Role of the Graphing Calculator in the Advanced Level Curriculum

While the new advanced level mathematics curriculum continues to emphasise mastery of mathematical concepts and acquisition of mathematical skills, greater emphasis is now given to the development of students' abilities to conjecture, discover, reason and communicate mathematics with the aid of technological tools. To achieve this objective, classroom practices and pedagogical strategies will require some adjustments on the part of the teachers. For instance, the graphing calculator will play a key role when students engage in stimulating discussions on, and activities in, mathematics where they can explore possibilities and make connections. Indeed, the use of graphing calculators is intended to be integral to the learning of mathematics at the advanced levels in the new curriculum.

Graphing calculators should be used in ways which allow students to learn mathematics in practical and meaningful context, using analytic methods together with graphical and numerical techniques. The computational and graphing capabilities of graphing calculators will enable students to engage in active learning through exploratory work and experiments. Students will work collaboratively with others, share ideas and discuss their findings. Graphing calculators will also extend the range of problems accessible to students and enable them to execute routine procedures quickly and accurately, to make connections between algebraic and geometric ideas, and to switch swiftly between numerical, graphical and symbolic representations in mathematical explorations. By and large, the routine tasks should be relegated to the graphing calculator, allowing more time for thinking, reflection and discovery in the new curriculum.

The incorporation of non-CAS graphing calculators in assessment presents even greater challenges for teachers and students alike. In the new syllabus, the examination papers will be set with the assumption that candidates will have access to graphing calculators. As a general rule, unsupported answers obtained from graphing calculators are not allowed and candidates are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, candidates are expected to sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using graphing calculators correctly, method marks may be awarded (MOE, 2005). Students are also expected to be aware of the limitations that are inherent in graphing calculators.

In a nutshell, in the new curriculum students need to be proficient in solving problems with the aid of a graphing calculator under conditions of a timed examination. As the understanding of concepts of functions is fundamental in the learning of mathematics, students are expected to use the graphing calculator to explore properties of graphs, determine the range of a function from the graph, and examine the conditions for the existence of inverse functions and composite functions. Apart from using the graphing calculator as a graphing tool to investigate behaviours of various types of functions (such as trigonometric, exponential and logarithmic functions) and the effects of transformations on these functions, students are also expected to know how to use a graphing calculator to solve problems on different topics in the syllabus. For example, students need to learn how to use a graphing calculator to solve inequalities by graphical methods, find the numerical solution of an equation, carry out addition, subtraction, multiplication and division of complex numbers, find the square roots, modulus and argument of a complex number, convert a complex number from Cartesian form to polar form and vice versa, evaluate values of the derivative of a

given function at given points, locate maximum and minimum points of a function, investigate the relationship between the graph of a given function and that of its derivative, derive the Maclaurin's series for a function and use the first few terms of the series to approximate the function, illustrate concepts of limits (particularly that of a Riemann sum), evaluate definite integral, and sketch typical members of the family of solution curves of a differential equation.

Given the wide range of skills pertaining to the use of graphing calculators students are required to master, teachers are expected to provide sufficient guidance to help the students develop accurate conceptual understanding of the graphing calculator and its functionality. To this end, it is imperative that training in the use of a graphing calculator with an emphasis on solving typical examination problems be provided to the teachers. Equipped with the basic skills in solving problems with the graphing calculator, the teachers can then be in a position to examine and adjust their classroom practices so as to optimise the educational benefits of integrating the use of a graphing calculator in the mathematics curriculum.

About the In-service Course

The in-service course, which is comprised of three 3-hour weekly sessions, aims to familiarise the participants with the basic functionality of the TI-84+ and equip them with the knowledge and skills necessary to solve advanced level mathematics problems with the aid of the calculator. A total of 90 teachers who are teaching mathematics at the advanced level have attended the in-service course. None of the teachers had used a graphing calculator before attending the course. A set of notes prepared by the author which contains explanations on the features of the TI-84+ and the keystrokes necessary to perform the operations covered in the course was given to each participant so that the participants could focus on the hands-on activities without having to take copious notes during the course. To facilitate demonstration of the functionality of the TI-84+, the TI Emulator was used as a main medium of instruction.

The first session familiarised the participants with the general functionality of the TI-84+ before basic as well as advanced graphing features of the calculator were introduced to them. A hands-on session then followed during which the participants explored the use of the TI-84+ in understanding concepts of functions and in investigating the conditions for the existence of inverse and composite functions. During the second session the participants were shown how the TI-84+ could be used to do mathematics in Calculus, Complex Numbers and Vectors. The participants were particularly impressed that although the TI-84+ does not have any provision for performing operations on vectors, by making use of the LIST function of the calculator, one could still compute the addition, subtraction, dot product and cross product of any two given vectors. In the third session, the participants were asked to apply the skills they had acquired in the first two sessions to answer questions in a collection of 26 typical advanced level examination questions covering the various topics in the advanced mathematics syllabus. The role which the graphing calculator could play in answering each of the questions was discussed.

Based on observations of the in-class work and the responses of the participants during the course, some of the examples given in the course were found to be particularly useful to the teachers. This paper shall share four such examples. In particular, we shall see how the TI-84+ can be used to solve equations interactively, express an algebraic fraction in partial fractions, explore the existence of composite functions and find the first few terms of a Maclaurin series. These examples were chosen either to demonstrate certain features of the TI-84+ that are particularly useful (such as the interactive Equation Solver), to highlight certain new focus in the mathematics curriculum following the incorporation of the graphing calculator in the syllabus (such as the existence of inverse and composite functions) or to highlight certain limitations of the calculator.

Usefulness of the Equation Solver

The Equation Solver is a very useful feature of the calculator. Given an equation with one or more variables, the Solver can solve for any variable if the values of the other variables are given. Participants of the in-service course were asked to explore the use of the Equation Solver in solving problems such as the following.

Example 1

ABC is a triangle in which $BC = 3$ cm, $CA = 7$ cm and angle $B = 60^\circ$. Find the exact value of $\cos C$.

It is easy to see that the Cosine Rule can be applied to solve this problem. To solve the first part of the problem, we enter the equation in the interactive Solver editor as shown in Figure 1, putting $A = 3$, $B = 7$ and $X = \frac{\pi}{3}$. We need to enter an initial guess for the variable C for which we are solving.

We can simply pick any positive number, say 5. Solving for C , the calculator returns $C = 8$ (see Figure 2).

```
B²-A²-C²+2ACc...=0
B=7
A=3
C=5
X=π/3
bound={-1E99,1...
```

Figure 1

```
B²-A²-C²+2ACc...=0
B=7
A=3
C=8.0000000000...
X=1.0471975511...
bound={-1E99,1...
left-rt=0
```

Figure 2

To solve the second part of the problem, we put $B = 8$, $A = 7$ and $C = 3$ and then solve for X . The calculator returns $X = 1.7141438957...$ (see Figure 3).

```
B²-A²-C²+2ACc...=0
B=8
A=7
C=3
X=1.7141438957...
bound={-1E99,1...
left-rt=2.6E-12
```

Figure 3

```
cos(X)►Frac -1/7
```

Figure 4

Notice that the above value is now stored in the variable X . To find the exact value of $\cos X$, we return to the HOME screen, enter $\cos(X)$ and convert the value of $\cos X$ to a fraction (see Figure 4).

Hence the exact value of $\cos X$ is $-\frac{1}{7}$.

The above activity illustrates that even though the TI-84+ cannot solve an equation for its exact root(s) with just a single command the way a CAS calculator can, there are more one can do in solving equations using the TI-84+ than settling for a numerical value of the root of the equation. From a pedagogical viewpoint, the teacher should discuss how the cosine rule is applicable in solving a problem such as that in Example 1, how the exact value of $\cos C$ can be obtained algebraically, and the significance of the value obtained. The graphing calculator could serve to verify the answer obtained and be used to demonstrate the relationship between the sides of the triangle and the unknown angles. The next example further demonstrates the usefulness of the above features that the calculator affords.

Example 2

Express $f(x) = \frac{7x^2 - 14x + 18}{(x-3)(x^2+4)}$ in partial fractions. Hence find the exact value of $\int_0^2 f(x) dx$, giving your answer in a form involving a natural logarithm and π .

Expressing the given algebraic fraction in partial fractions entails solving for A, B and C in the identity $7x^2 - 14x + 18 = A(x^2 + 4) + (Bx + C)(x - 3)$ as shown in Figure 5. To solve for A, we put $x = 3$, enter any number for B and for C in the interactive Solver editor (in Figure 6, we enter $B = 1$ and $C = 1$). The calculator promptly returns $A = 3$. We next obtain $C = -2$ by putting $X = 0$ and leaving $A = 3$ as obtained earlier. Note that since we put $X = 0$, we can put B to be any value. In Figure 7, we leave $B = 1$ as entered previously. Finally, putting $X = 1$ and leaving $A = 3$ and $C = -2$ as they are, we obtain $B = 4$ (see Figure 8).

```

EQUATION SOLVER
eqn:0=(7X^2-14X+18)-A(X^2+4)-(BX+C)(X-3)
  
```

Figure 5

```

(7X^2-14X+18)-...=0
X=3
A=3
B=1
C=1
bound=(-1E99,1...
left-rt=0
  
```

Figure 6

```

(7X^2-14X+18)-...=0
X=0
A=3
B=1
C=-2
bound=(-1E99,1...
left-rt=0
  
```

Figure 7

```

(7X^2-14X+18)-...=0
X=1
A=3
B=4
C=-2
bound=(-1E99,1...
left-rt=0
  
```

Figure 8

Now, since $\int_0^2 f(x) dx = \int_0^2 \frac{3}{x-3} dx + \int_0^2 \frac{4x}{x^2+4} dx - \int_0^2 \frac{2}{x^2+4} dx$, to find the exact value of the definite

integral $\int_0^2 f(x) dx$ we evaluate separately the three definite integrals on the right-hand side of the

above equality using the numerical integration function of the TI-84+. Entering **fnInt(3/(X-3),X,0,2)**, the calculator returns the value -3.295836866 which is now stored in the variable **Ans**. Expecting this value to be in a form involving a natural logarithm, we enter the expression **e^(abs(Ans))** and the calculator returns the value 27 (see Figure 9). It follows that

$\int_0^2 \frac{3}{x-3} dx = -\ln 27$. Likewise, entering **fnInt(4X/(X^2+4),X,0,2)** followed by **e^(abs(Ans))**, we

obtain the value 4 (see Figure 10). It follows that $\int_0^2 \frac{4x}{x^2+4} dx = \ln 4$. Finally, we enter

fnInt(2/(X^2+4),X,0,2). Expecting the answer to be in a form involving π , we enter the expression as shown in Figure 11. The calculator returns the value $\frac{1}{4}$ (see Figure 11). Hence, we obtain

$$\int_0^2 \frac{2}{x^2+4} dx = \frac{\pi}{4}. \text{ Therefore, } \int_0^2 f(x) dx = \ln \frac{4}{27} + \frac{\pi}{4}.$$

```

fnInt(3/(X-3),X,
0,2)
-3.295836866
e^(abs(Ans))
27

```

Figure 9

```

fnInt(4X/(X^2+4),
X,0,2)
1.386294361
e^(abs(Ans))
4

```

Figure 10

```

fnInt(2/(X^2+4),X
,0,2)
.7853981634
Ans/π*Frac
1/4

```

Figure 11

Exploring the Existence of a Composite Function

The graphing calculator can be used to investigate the condition for which a composite function exists. The following example was used for such an investigation in the in-service course.

Example 3

Functions f and g are defined by

$$f: x \mapsto x^2 + 1; x < 1,$$

$$g: x \mapsto 3x - 5; x \geq 1.$$

Investigate the conditions for the existence of the composite functions $f \circ g$ and $g \circ f$.

We first enter the functions f and g as Y_1 and Y_2 as shown in Figure 12 and then graph the two functions. We can then determine graphically that the ranges of f and g are given by $R_f = [1, \infty)$ and $R_g = [-2, \infty)$ respectively (see Figure 13).

```

Plot1 Plot2 Plot3
\Y1=(X^2+1)/(X<1)
\Y2=(3X-5)/(X≥1)
\Y3=
\Y4=
\Y5=

```

Figure 12

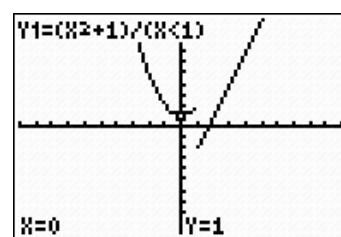


Figure 13

Since R_f equals the domain of g , we deduce that $g \circ f$ exists. Alternatively, we define $Y_3 = Y_2(Y_1(X))$ and deselect Y_2 as shown in Figure 14, and then graph Y_1 and Y_3 on a suitable window (see Figure 15). We can see that the graphs defined by Y_1 and Y_3 have the same domain. Therefore, the composite function $g \circ f$ exists.

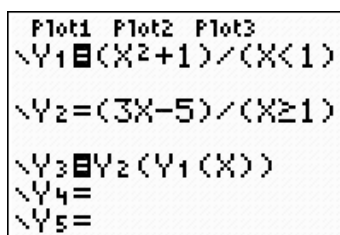


Figure 14

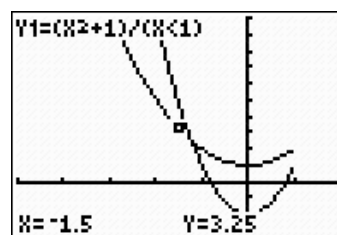


Figure 15

The rule of the composite function $g \circ f$ can be determined algebraically to be that given by $g \circ f(x) = 3(x^2+1) - 5 = 3x^2 - 2$. Using the TI-84+, we can verify graphically that the rule of $g \circ f$ is indeed as obtained above by defining Y_4 as shown in Figure 16 and then graphing Y_3 followed by Y_4 while Y_1 and Y_2 are deselected. To create the visual effect that Y_3 ‘coincides’ with Y_4 , we can graph Y_3 in a thin line and Y_4 a thick line in that order (see Figure 17).

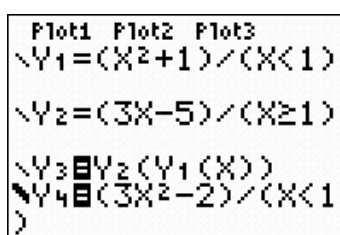


Figure 16

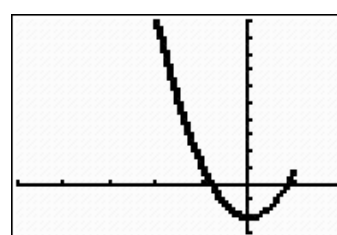


Figure 17

On the other hand, since f is only defined for $x < 1$, the range of g , namely R_g , is not a subset of the domain of f and so $f \circ g$ does not exist. Again, we can determine the existence of $f \circ g$ with the help of the calculator by defining $Y_5 = Y_1(Y_2(X))$ and then graphing only Y_2 and Y_5 (see Figure 18).

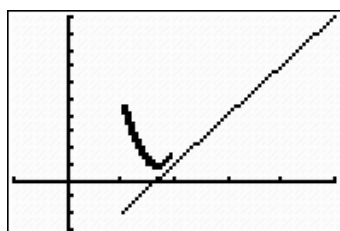


Figure 18

Obviously, the function entered as $Y_1 Y_2$ and that entered as Y_2 do not have the same domain. Hence, we can again conclude that the composite function $f \circ g$ does not exist.

Finding the Maclaurin Series

Although non-CAS graphing calculators such as the TI-84+ is not able to perform symbolic manipulations and computations, by applying the numerical derivative feature of the calculator we can still find the first few terms of the Maclaurin series, if exists, for a given function. The following example was used for discussion in the in-service course. Through this discussion, participants were also led to discover some limitations of the calculator.

Example 4

Find the first four terms of the Maclaurin series for $\ln(2+x)$. Write down the first four terms of the series for $\ln(2-x)$, and hence show that, if x is small, then $\ln\left(\frac{2+x}{2-x}\right) \approx x + \frac{1}{12}x^3$.

Show on a sketch the shape of the graph of $y = \ln\left(\frac{2+x}{2-x}\right)$ for small x , indicating clearly the relationship of the graph to that of $y = x$.

We first define Y_1, Y_2, Y_3 and Y_4 as shown in Figure 19. In the Home screen we then evaluate $Y_1(0)$ and $Y_2(0)$ (see Figure 20). Note that we expect the value of $Y_1(0)$ to be in a form involving natural logarithmic. So since the value 2 is returned after the expression $e^{(\text{Ans})}$ is entered, we deduce that the first term of the series is $\ln 2$. For $Y_2(0)$, in spite of the attempt to convert its value to a fraction, the calculator returns the value 0.5000000417 instead of $\frac{1}{2}$.

```

Plot1 Plot2 Plot3
\Y1ln(2+X)
\Y2lnDeriv(Y1,X,
X)
\Y3lnDeriv(Y2,X,
X)
\Y4lnDeriv(Y3,X,
X)

```

Figure 19

```

Y1(0)
.6931471806
e^(Ans)
2
Y2(0)Frac
.5000000417

```

Figure 20

Likewise, the calculator returns the value -0.25000025 instead of $-\frac{1}{4}$ for $Y_3(0)$. This is a limitation of the calculator which teachers need to point out to their students when using the calculator in solving problems. On the other hand, executing $Y_4(0)$ yields an error message (see Figure 21).

```

ERR:ILLEGAL NEST
Quit
Z:Goto

```

Figure 21

```

Plot1 Plot2 Plot3
\Y11/(2+X)
\Y2lnDeriv(Y1,X,
X)
\Y3lnDeriv(Y2,X,
X)
\Y4=
\Y5=

```

Figure 22

To overcome this problem, we replace the function $\ln(2+x)$, which was entered as Y_1 previously, by its derivative, namely $\frac{1}{2+x}$ (see Figure 22), and evaluate $Y_3(0)$ in the HOME screen. We again note that the calculator returns the value 0.25000025 instead of $\frac{1}{4}$ (see Figure 23).

```

Y3(0)Frac
.25000025

```

Figure 23

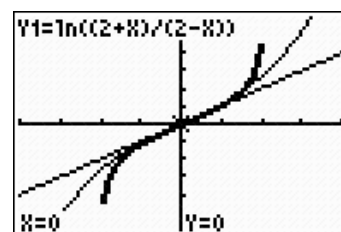


Figure 24

With the help of the TI-84+, we can do much more than the question requires us to in the last part of the problem. We can graph the functions given by $y = \ln\left(\frac{2+x}{2-x}\right)$, $y = x + \frac{1}{12}x^3$ and $y = x$ not just for small x , but for a much larger range of values of x (see Figure 24). This illustrates one obvious

advantage of introducing the graphing calculator in the classroom, for without which graphing functions such as $y = \ln\left(\frac{2+x}{2-x}\right)$ and $y = x + \frac{1}{12}x^3$ can be a rather tedious process.

Course Evaluation

At the end of the 9-hour in-service course, the participants were administered an evaluation form comprising two sections. The first section, which contains 8 Likert-type items and two open-ended items, serves to elicit participants' overall comments on the course. The second section, which is composed of 5 Likert-type items, is intended to obtain feedback for the trainer. The combined results of the course evaluations (for the 3 classes of 90 participants) are presented in Table 1 below. Only 6 of the 13 Likert-type items in the evaluation form administered are presented in the table as the other items are pertaining to administrative and logistical matters and are hence not relevant to our discussions here.

Table 1: Results of the Course Evaluations

	Strongly Agree	Agree	Others*	Total
The course objectives was achieved	46	44	0	90
The course materials were relevant to the objectives	56	34	0	90
I can apply the ideas/skills learnt	38	52	0	90
I will recommend the course to the others	42	48	0	90
The overall course presentation was good	37	53	0	90
The content met my learning needs	40	50	0	90

*Includes 'Disagree' and 'Strongly Disagree'.

All the participants agreed or strongly agreed (with at least 41% of the 90 participants strongly agreed) that the course had met the objectives of getting them started with the TI-84+ and training them in solving advanced level problems with the aid of the graphing calculator. They were particularly pleased with the set of notes given to them which they had found "very useful and helpful" during the course and would make "a very useful quick reference" in their teaching. It is also evident that the participants were impressed with the emulator software used by the author throughout the course, describing it as a "very useful visual aid". All the participants agreed that the course met their learning needs in that they were "more confident" in using the TI-84+ for teaching purposes after they attended the course.

Strengths of the course identified by many participants include the adoption of the discovery approach for the hands-on activities during which the participants could "construct [their] own learning". Describing this course as "practical" and which provided "many relevant examples, especially the typical advanced level problems," and being "tailored to [their] needs", it is clear that the participants found the hands-on activities with a focus on solving advanced level problems with the aid of the graphing calculator very useful. This generally reflects teachers' concerns about the incorporation of graphing calculators in assessment; the immediate concern of the teachers is to impart to their students the relevant skills in answering advanced level questions with the aid of a graphing calculator in assessments. It is therefore paramount that the teachers have a good knowledge of what the TI-84+ is capable of doing in the context of advanced level mathematics so as to plan their lessons accordingly to integrate the use of the graphing calculator into the classroom.

Conclusions

Following the incorporation of graphing calculators in assessment in the new advanced level mathematics curriculum, mathematics teachers in Singapore who are teaching at the advanced level now face the challenge of not only preparing their students in the content area but also training them in using the graphing calculator. Indeed, this is the task now faced by many mathematics educators in many parts of the world (Dunham & Dick, 1994).

In this paper the author has discussed the role which graphing calculators could play in the new curriculum, shared a few examples on how we can do more than just the basics of using the TI-84+ by exploring the use of several useful features that the calculator affords, and reported on the results of the participants' evaluation of the course. As we have seen, there are a lot more mathematics one could do with a non-CAS calculator than it seems. The limitations of the TI-84+ in several aspects have also been highlighted. Teachers should be aware of these limitations and their implications on students' learning experiences with the calculator. This further underscores the importance of preparing the teachers adequately in the integration of graphing calculators in the mathematics curriculum.

Undoubtedly, effective professional development, as pointed out by Waits and Demana (2000), is an important factor that positively influences the use of technology in the classroom. In addition to courses for teachers which focus on solving problems with the aid of graphing calculators, there should also be in-service training which addresses pedagogical issues specifically. To complement studies, such as that by Ng (2003), which examine the effects of CAS-calculator use, studies should also be conducted to investigate the impact of the use of non-CAS calculators on the teaching and learning of mathematics in Singapore to add to our knowledge of the educational benefits and potential of using graphing calculators in mathematics education.

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