An Integrated Learning Environment for Developing "Function Sense"(2)

-From Velocity to First Steps in Calculus Using Spreadsheets-

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Abstract: We would like to present a pilot study for exploring how to develop a "function sense" in an integrated learning environment (Kakihana *et al.*, 2000) with spreadsheets and experiments using a graphing calculator. In this pilot study, high school students and college students were examined to see how they used their "function sense" so that they could better understand the relation between a velocity and the concept of differential calculus. At first, they tried to understand the velocity and the shape of a function using a graphing calculator while actually walking. Then, they simulated many walking situations in an integrated environment using spreadsheets. They input the walking data (velocity or distance) or the algebraic expression of the velocity of this duration. Graphs of the distance and velocity were drawn automatically. On the worksheets, students were asked what kinds of relations were shown on the graphs, the inputted numeric data, the walking situations and the concept of velocity. As the result of these activities students were able to correlate the walking situations and the velocity. These experiences, we believe, will lead students to effectively understand calculus.

1. Background

1.1 First steps in calculus.

In high school calculus, the concept of a rate of instantaneous change is used as a derivative of a function. In textbooks, a derivative of a function is defined as follows:

> A derivative of function *f* at point *a* is the limit of the slope of the secant line through (*a, f(a)*) and any other point (*x, f(x)*) on the graph as *x* approaches *a,* that is the slope of the tangent line to the graph at *(a, f(a)*).

Formulas are given to calculate a derivative for many kinds of functions and students practice these calculations without understanding the concept of a derivative. Many students feel it is boring it and so they cannot understand these definitions.

On the other hand, students enjoy the activity “Let’s draw a graph by walking”, in which they try to walk at various velocities and by means of a graphing calculator. In this way, they can understand the relationship between the shape of the graphs and velocities. (Saeki, A. *et al*., 1997)

A function has two aspects, one is to express “a correspondence of two values”, and the other is to express “a change”. It seems that the first one is stressed more than the second one in mathematics education in Japan. Hitotsumatsu. said, "It is more important to learn concepts of functions as the expression of change than as concepts of correspondences, especially in calculus." (Hitotsumatsu, 1998).

Student need to make a connection between the concept of calculus and the walking activity.

1.2 Function and its learning environment.

We reported that it's very important for students to be able to use many kinds of functions to express a change in the real world and to cultivate their "function sense" for it. "Function sense" is an intersection of three senses; numerical sense to connect numeric data with patterns of tables or graphs, visual sense to connect a graph with patterns of algebraic expressions or data and algebraic sense to connect algebraic expressions with patterns of graphs or data (Kakihana *et al*., 2000). For cultivating this sense, we have studied the integrated learning environment in which numerical data, a graph and an algebraic expression of function are used interactively and visually as tools to solve a problem (Fukuda, *et al*., 2001). This environment is very effective for learning functions (Fukuda, *et al*., 2002).

With that background, we planned to connect walking with simulating speeds and distance on a spreadsheet with visually understanding the concept of a derivative.

2. Purpose

(1) To develop the teaching materials to cultivate "Function sense" from the viewpoint which emphasizes the rate of change of functions.

(2) To evaluate the simulation system which was developed for the above purpose.

3. Methods

Subjects: Seven 17 year old high school students (two males, two females and a group of three females), who had never learned calculus. And two college students who had learned calculus in the high school.

Process and Task:

1) Students explained the velocity and the shape of a function with a graphing calculator by actually walking.
   Task 1: Nine graphs are given. Make those graphs by walking.
2) They simulated many walking situations by inputting numeric velocity or distance data.
   Task 2: Draw a time-distance or time-velocity graph of a verbalized situation (constant change rate).
   Task 3: Draw a time-distance or time-velocity graph of a verbalized situation (changing the change rate).
   Task 4: Draw a time-distance or time-velocity graph of a situation expressed on a graph
   Task 5: Draw a graph from a given algebraic expression.
3) In an examination, they were told to connect the distance graph with the velocity graph without technology.
   Comprehension-check 1: A verbalized situation
   Comprehension-check 2: A situation expressed on a graph

The design of an experiment: This time, we considered six elements included not only functions but also derivatives, and designed our experiments in which students were able to go back and forth those elements. Those six elements are a set of value, an algebraic expression and a graph for algebraic expression of a distance or a velocity (Fig. 1). For example, in the Fig.1, arrows show the flow of the task 5-I and task 5-II.

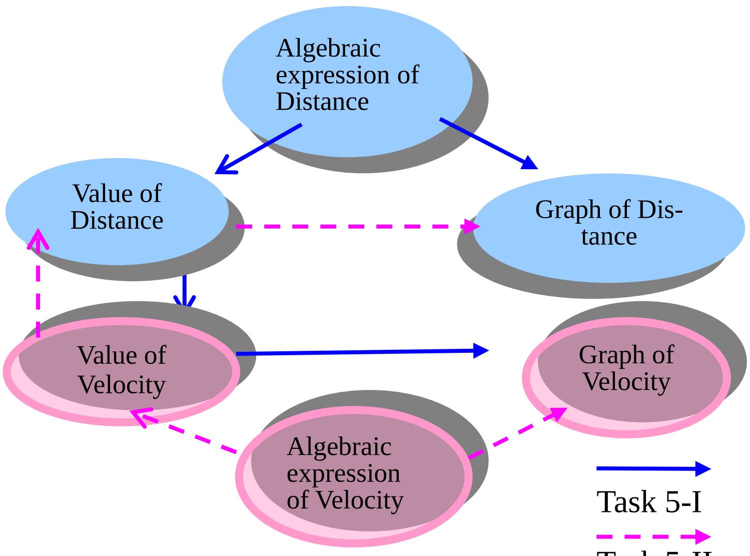


Fig. 1 The design of an experiment

An integrated learning environment: The simulation system (Fig. 2) relies on Microsoft Office Excel 2003. First, set the time unit and a starting point. Then, if students input distance per one time unit or velocity in the sheet, distance from the starting point is calculated automatically and the distance graph appears. If students input distance from the starting point in the sheet, distance per one time unit or velocity is calculated automatically and the velocity graph appears.

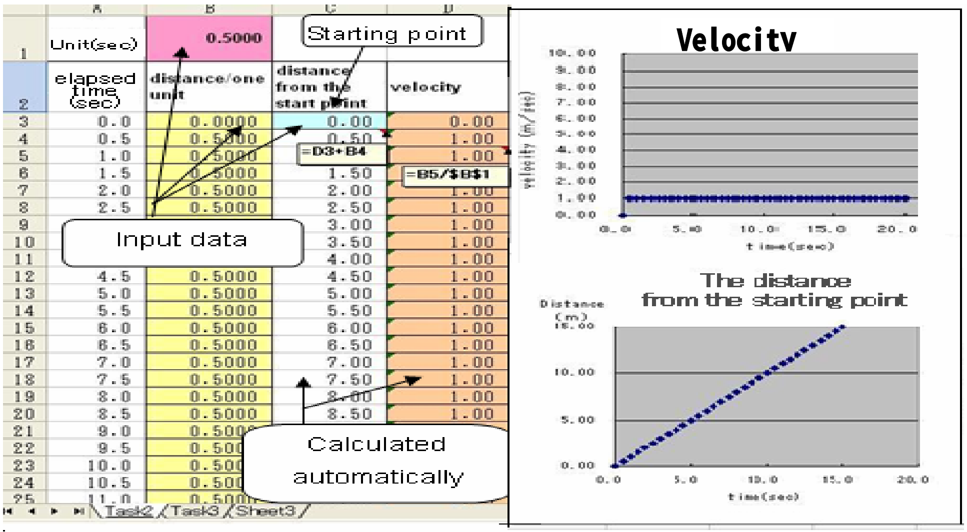


Fig. 2 Simulation system (Example of Task 2)

4. Result of simulation

4.1 Simulating a verbalized situation

1) The case of a constant change rate (Task 2):

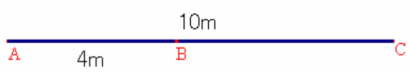
<u>Task 2</u> (Excerpt from original)

Let 's think about time-distance graph in various situations. First, imagine the graph and draw the graph in the rectangle below. Next, input the distance from start point to point A, then input the distance per unit time consecutively. You can see the graph in the spreadsheet.

your conjecture

Situation 1

I start at point A and walk to point C at a constant velocity.
Is the graph the same as your conjecture?
How do you input this in the spreadsheet?
What kind of graph can you see?
Describe the characteristic of the graph.



Because they had the experience of making a linear function graph by actually walking, they were immediately able to input the numerical value of the distance moved during the time unit (Fig. 2). Although we expected that it would be difficult for them to input negative values for distance moved, they were able to input them when they confirmed them on the graph. In the beginning, they were told to input the distance moved in the time unit 0.5 seconds and next the distance per second (velocity). We worded the question in such a way that they could clearly distinguish the rate of change from the actual distance moved and so that they could notice the rate of change.

2) The case of varying the change rate (Task 3):

<u>Task 3</u> (Excerpt from original)

A train started from station A, and it arrived at station C in 6 minutes. On its way, it stopped at station B for 30 seconds. Let's think about the distance from station A to the train, and imagine the time-distance graph. Then input the distance per unit time.
Compare the graph on the spreadsheet with your conjecture.

At first, the student graphed the situation in which the train left in a straight line. When we pointed out that the train accelerated gradually, they could reproduce it. One student did not take into account the value of the change rate. Therefore, the distance grew too much, and he suddenly adjusted it during the task (Fig. 3).

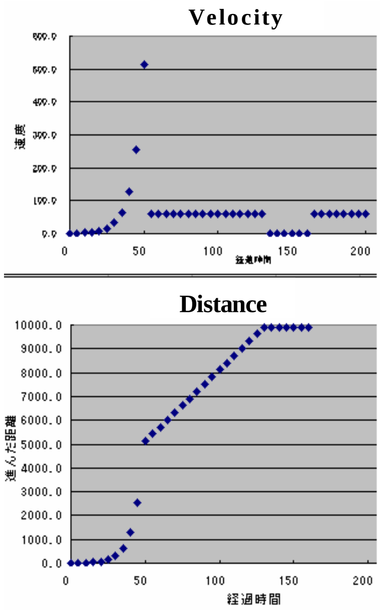


Fig. 3 Task 3

4.2 Simulating a situation expressed on a graph

Task 4

I. Look at these time-distance graphs (fig. 1, fig. 2). First, imagine the velocity in this situation and input zero to the start point, then input the velocity per unit time consecutively. Did you draw the same graph as fig. 1 or fig. 2?

II. Look at these time-velocity graphs (fig. 3, fig. 4). First, imagine the distance in this situation and input the distance from the start point to the walker, then input the distance per time unit consecutively. Did you draw the same graph as fig. 3 or fig. 4?

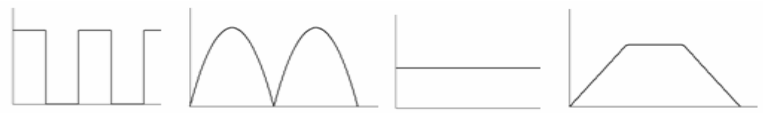


fig. 1 fig. 2 fig. 3 fig. 4

1) Draw a velocity graph by inputting distance (Task 4-I): After having seen the distance graph, the students were told to input the numerical value of velocity on the sheet, and make the graph of the velocity and the distance. In Task 4-I, there was no change at the start but some students put the velocity as a positive number. They were tempted to input a positive value because the given distance graph took a positive value (Fig. 4). The 0 change rate was not easily noticed.

Student K got a hint from the situation in Task 3 in which the train had left, input the numerical value, and completed the graph (Fig. 5). However, at first, some students had input the velocity as

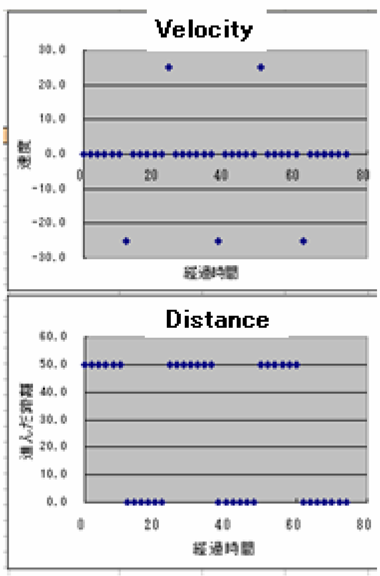


Fig. 4 Task 4-I (a)

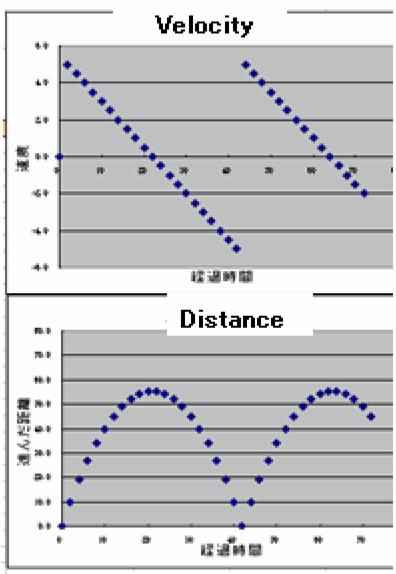


Fig. 5 Task 4-I (Student K)

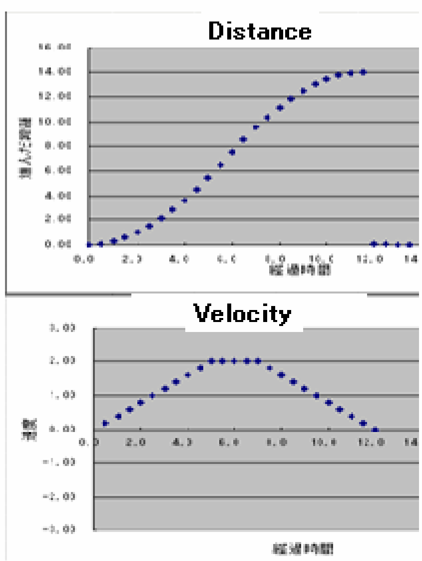


Fig. 6 Task 4-II

1,2,3... so they were not able to construct the graph that they expected to make. When the shape of points in the distance graph goes the upper right, the students enlarged the change rate. Since we advised them to imagine walking or to remember the numerical input of a previous topic, they checked the graph and noticed that the rate of change decreased.

In that stage, the rate of change which students input was varied, linear, exponential, etc., and they had paid little attention to the regularity of a rate of change.

2) Draw a distance graph by inputting velocity (Task 4-II): In trying to make the velocity graph which they desired, it was difficult to input distance. Some students interpreted the state where velocity was fixed as having stopped, and confused reduction in velocity with reduction in distance. This task was more difficult for students than we expected (Fig. 6).

4.3 Simulating the situation expressed formula

1) Draw a distance graph by inputting velocity (Task 5-I): This task required inputting velocity and drawing the distance graph of the formula $S = 0.2t^2$. Although they were dealing with the distance graph which drew a parabola in a previous problem, the students were not able to input a numerical value easily. We proposed changing a procedure, making the graph of $S = 0.2t^2$ using a spreadsheet and focusing on the difference. The students were surprised that the graph of difference becomes a straight line from the negative to positive value exceeding 0.

2) Draw a velocity graph by inputting distance (Task 5-II): Formula of velocity $V = 2t$ was given; they were drawing a distance graph which was increasing gradually. However, some students were not conscious of it becoming a parabola.

We summarized the activity up to that point and checked the relation between the distance per time unit, velocity, and the shape of the graph.

5. Results of the confirmation test

Finding the rate of change without technology.

5.1 A verbalized situation
(Comprehension check-1)

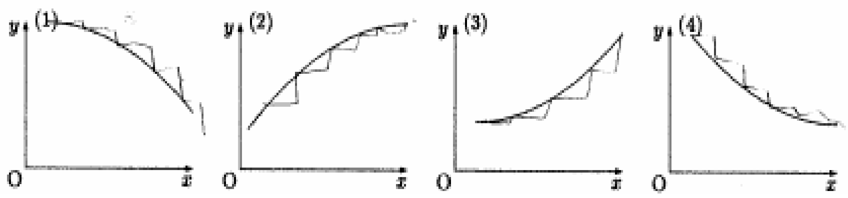
We then checked whether students noticed a change rate in the graph. For example, “When the mark goes to the right by one and then up by one, a change rate is positive”, “When it goes down by one, it’s negative.”, or “When it fell significantly, the absolute value becomes large.” They calculated the average rate of change of the time unit and then connected the situation with its graph (see Comprehension check -1).

Comprehension check-1 (Excerpt from original)

Graphs (1) to (5) below show the populations of certain cities for the last thirty years. Graphs (a) to (f) below show their rate of change. Let’s read illustrations below and choose the corresponding graph

(a) “At first the population of city A increased, but the rate of increase was slow and then gradually decreased.”
Choose the graph corresponds to city A’s population.
Choose the graph corresponds to the rate of change.

(b) “The population of city B increased, but recently the rate of increase is gradually slow.”
Choose the graph corresponds to city B’s population.
Choose the graph corresponds to the rate of change.



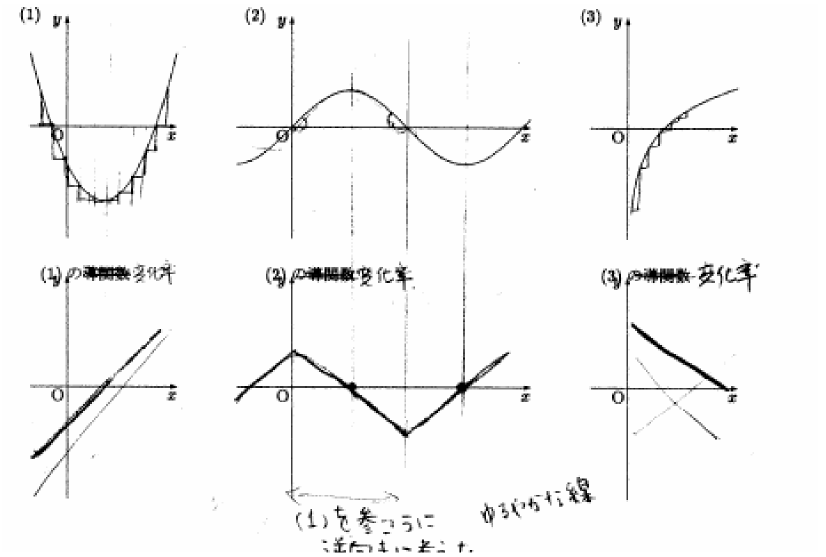
5.2 A situation expressed in a graph (Comprehension check-2)

This is a task in which students read a rate of change in the given graph of a function, or predict a function from a rate of change without using technology.

About graph (1), the shape of a linear function was made since they considered this kind of graph in Task 5-I using technology. Although they did not pay attention to its position. (See comprehension check-2) Graph (3) had the same tendency as Task 4-I and the teacher called attention to it. Student K considered graph (2) to be the combination of graph (1) and drew it.
Even here, some students were allured by the form of the function graph and mistakenly found the rate of change intuitively. They noticed the error if a teacher urged caution in considering a rate of change (Fig. 7).

Comprehension check-2 (Excerpt from original)

Draw graphs of each function and its derivative.



6. Discussion

1) The advantage of actually experiencing and simulating the situation using a numerical value.

By experiment in Task 1 "Let's walk and draw graph", correlation of shape of graph with velocity could actually be experienced. Also, when students considered the simulation in the subsequent tasks, we were able to propose that they remember their walking experience when they didn't understand anymore (in the case of Task 4). By recalling their experience, they did not miss the meaning of the simulation. However, another simulation involved a scenario which was beyond their ability to actually experience. This scenario is shown below.

- Task 3 (Fig. 3): It is difficult to actually move by a rapid change of distance as shown in Task 3 (Fig. 3). However, they have reproduced this scenario in the simulation.

It wasn't easy for students to understand the effect of rapid changes. Student A said, "Although we cannot actually walk like this, we can do it on software". This will strongly motivate students to consider the continuity or differentiability of a function in the future.

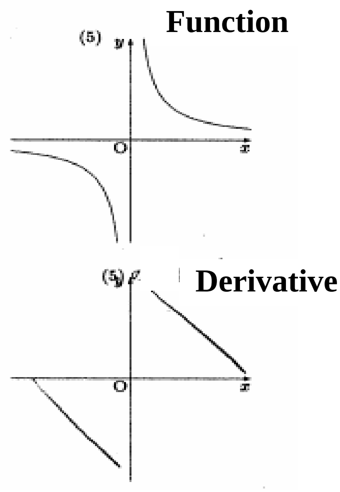


Fig. 7 An example of misunderstanding

- Although it was unnatural to have considered the negative value of x (time) by actually walking, in the simulation students were able to consider the negative x automatically just like the positive value of x.

- When we walk, it is relatively possible to move fast or slowly, but it is difficult to walk at twice that speed. Therefore, it is hard to grasp change of velocity or distance as a quantity.

If we calculated the difference by using the numerical value from CBL, this raw data is too unclear to allow students to find the regularity in it. However, this method, by which we can anticipate the relation in an experiment and simulate it, expresses the rate of change as digital data and makes it easy to shift to the formula in the following..

2) Possibilities from exploring change rate to introduction of differentiation

In the development stage of these teaching materials, we tried to test the materials on college students who have already learned calculus. College student M said, “I use this formula for the differentiation of the polynomial of degree n and I use that formula for the differentiation of sin. I considered them separately, but then I felt that they were connected as one.” Student N said, “I got it. I had memorized that the velocity was equal to the differentiation, the differentiation was calculated by a formula and this kind of problem was solved by the differentiation, but if I understand the meaning, I will apply it more effectively.”

Thomas W. Judson and Toshiyuki Nishimori (2003) compared Japanese and American high school students' concept of and skills in calculus. They determined that a Japanese high school student's algebra skill is very high. However, the question is whether an understanding of a concept is progressing. From student S we heard, “We feel these teaching materials urged a conceptual understanding.”

3) About this simulation system

If students input numerically the quantity which changed with the time unit, the velocity graph will change, the amount of change will be accumulated further and the numerical value of distance and the graph will change. This system allowed various interpretations, while the students went back and forth with some expressions (fig. 2). These teaching materials were created using only the fundamental function of a spreadsheet. Students used it, without being conscious of technology, and there was no scenario where thinking was interrupted by technology operations. Moreover, it was easy for the teacher to maintain it, and it was possible to customize it for his lesson.

4) Confusion of increase / decrease of a function and increase / decrease of a change rate

In Task 4 or the evaluation test, all students frequently confused the change in a function with the change in a change rate. When the function was decreasing, a rate of change only became negative but it did not necessarily decrease. However, one student drew a graph in which a rate of change also decreases. They did not differentiate between a change of a function, nor the change of a change rate. It was also the same when a function increased. When we advised that they improve the numerical value of the distance per time unit or a rate of change in a table, they noticed the error. They did not tend to abandon this kind of misconception easily so a teacher has to consider this point very carefully. Moreover, also in order to avoid this misconception, it can be said that the simulation which expresses the quantity of change numerically is effective.

7. Conclusions

1. This environment helped students to understand the relation between the distance and the velocity on a graph through inputting numeric data.
2. On the spreadsheet students can experience the situation which they are not able to activate actually in a real world
3. Students were able to connect the velocity graph with the distance graph without technology af-

ter activities on the simulation.
4. These activities were able to motivate students to learn calculus.

This study is taken from a paper by Fukuda and Kakihana (2004) which was presented at a conference of the Japan Society of Mathematical Education (2004) and another Kakihana and Fukuda paper (2004) which was presented in the poster sessions of ATCM 2004.

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