Connecting Geometry, Algebra and Calculus with The Geometer's Sketchpad (GSP): Thailand Perspective

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Abstract

The purpose of this study is to explore the use of the Geometer's Sketchpad (GSP) as a tool in connecting geometry, algebra and calculus in secondary schools in Thailand. GSP is one of the dynamic mathematics software that provides opportunities for teachers and students to use their ability to access, drag, visualize, and create graphical representation. GSP was translated into Thai language and used widely in Thailand.

In the 2005 school year, action researches were conducted in secondary schools in Thailand. The research findings show that Thai GSP play an important role in providing the representation and mathematics modeling. The students were able to express their understanding of mathematics concepts by verbal interactions and by constructing graphs. The students' works showed their creativity and their understanding of the mathematics problems.

Introduction

Geometry algebra and calculus are important subjects in mathematics. In order to success in these subjects, students have to learn and understand the concepts and symbolisms, know how to construct *representation* in mathematics, including grasp basic algebraic concepts before they study pre-calculus, linear algebra and other more advanced areas of mathematics. Accord to Cuoco and Curcio (2001) the representation is the map. Representations are packages that assign objects and their transformation to other objects and their transformations. The representations capture the process of constructing a mathematical concept or relationship. Representations allow mathematicians to record, reflect on, and later to recall their process and thinking. The representations may take a variety forms, from diagrams and models to graphs and symbolic expressions. The mapping diagram representation highlights some important features of functions that are often hidden by symbolic, tabular and formulas. In problem solving, a diagram can serve to create the structure of a problem and lay foundation for its solution. Meyer (2001) explains that students bridge the gap between concrete and increasingly abstract levels through their creation and use of models, drawings, diagrams, tables or symbolic notations.

However, calculus is one of the subjects that most students learn without understanding. The reason might be that with calculus, there has traditionally been an overwhelming emphasis on the manipulation of symbols, notion of derivative and the fundamental theorem of calculus. Because of that it has been too abstract for many students. Teachers expect their students to spend large amounts of time attempting to paper-and-pencil algorithms. Students are required to memorize symbolic computational techniques and formulas, but they do not really understand and quickly forget them (Sierpinska, 1994; Skemp, 1978).

We, mathematics educators, have to sought ways of supporting students in developing a mathematical understanding of all representations and symbolic by making the connections between expressions and graphs as transparent as possible. In addition, we need to teach our students to think critically and to do mathematics as mathematicians do. Students need many mathematical abilities to solve problems. These abilities include being able to detect patterns, conjecture, make generalizations, perform inductive and deductive reasoning, and make analogies. These skills can be nurtured through solving problems, engaging in exploratory investigations, experiencing classroom discourse, and using information and communication technology.

Nowadays there are various information and communication technology which can be used effectively in mathematics class. One of them is a dynamic mathematics software named *the Geometer's Sketchpad (GSP).* GSP empower students to use their ability to access, drag, visualize, and create graphical representation, which will enable them to develop their mathematical thinking skills, concepts and understanding. GSP provides opportunities for students to investigate and discover mathematics concepts in particular geometry algebra and calculus concepts. By using GSP students learn through exploring, investigating and discovering.

The Geometer's Sketchpad and Action Research in Thailand

The Geometer's Sketchpad (GSP) was introduced in Thailand since the year 2000, in year 2004 GSP was translated into Thai language and used widely in Thailand. More than 1,000 mathematics teachers were trained to use GSP as a tool in their mathematics classes. At least 100 workshops on the use of GSP were conducted by various agency in Thailand such as SEAMEO RECSAM (Southeast Asian Ministry of Education Organization, Regional Centre for Science And Mathematics), Ministry of Education Thailand under World Bank loan project on "*Secondary Education Quality Improvement on Mathematics (SEQI): Module 3*, The Institute for the Promotion of Teaching Science and Technology (IPST), Rajabhat Universities, Chiangmai University, and Prince of Songkhla University.

Action Research Questions

After attending the GSP workshops, many teachers consequently conducted action research in mathematics incorporate with the use of GSP as a tool in their classes. In Thailand, the students learned mathematics 5 hours per week and there are 16 weeks in one semester. The mathematics teachers face a challenge in balancing their teaching method on traditional chalk-and-talk "telling approach" and paper-and-pencil incorporating the use of GSP as a tool in their mathematics class and in completing the mathematics contents in the required syllabus. They are also face a challenge in turning mathematics into an interesting subject in order to attract more students to like mathematics.

Since GSP is a new tool for teachers to use in their mathematics class and the duration of this study is in the first semester of the school year, so it is not suitable to conduct the experimental research and compare the students' achievement between the students who use GSP and those who did not use GSP in mathematics class. With all this reasons, the action researches focused on using GSP to create representation in mathematics and using GSP to help students explore mathematics concepts with a balance use of paper-and-pencil.

The action research questions were:

- In what way GSP can be used as a tool in mathematics classes?
- What are the effects of GSP towards students' attitudes in mathematics?

Samples

The action researches were carried out in School year 2005 in many schools in Thailand. Ten mathematics teachers who attended GSP workshops of SEQI project: Ministry of Education Thailand are the sample group of this study. They are teachers of Suankulab Wittayalai school, Bangkok Thailand, Kannasoot Suksalai school, Supanburee Thailand, and Nong-Chang Secondary school Uthaithanee, Thailand.

Connecting Geometry Algebra and Calculus with the Geometer's Sketchpad

As described earlier, the action researches were conducted in schools in Thailand. The students learned mathematics on traditional approach "paper-and-pencil" incorporated with the use of GSP as a tool in their mathematics class. The following examples show lessons and activities in connecting geometry algebra and calculus with the Geometer's Sketchpad which were used in the action researches in Thailand.

Example 1: Exploring Derivative Functions

The derivative functions are one of the topics in calculus in secondary mathematics curriculum in Thailand. The teachers in the action research group revealed that in the previous year they found out that most of their students did not really understand this topic even though it was explained clearly step-by-step. For this semester, the teachers changed their teaching methods by using GSP together with their explanation of this topic to whole class. Then students studied this topic again in the computer lab.

The following activity shows how teachers used GSP as a tool in connecting algebra and calculus in their mathematics class. Students used GSP to drag and create graphical representation in order to develop their mathematical thinking skills, concepts and understanding. Students explored the derivative of a function: *A Tangent Line*. The following activities were used to enhance students to get more acquainted with the derivative of a function.

(1) Start the Geometer's Sketchpad program and choose *New Sketch* from *File* menu, then construct graph of function $f(x) = ax^3 + bx^2 + cx + d$ and the secant line as follows:

- Construct parameter *a, b, c* and *d* or use sliders from Custom Tool
- Enter function $f(x) = ax^3 + bx^2 + cx + d$ and plot graph of its function.
- Construct ***one point*** anywhere on the function plot ***f***. Label as point A and measure its abscissa (X_A) and ordinate (Y_A). Drag point A to observe how you can vary X_A and Y_A ;
- Construct parameter or slider ***h;***
- Choose *Calculate* from the *Measure menu* and calculate $X_A + h$, and $f(X_A + h)$;
- Select $X_A + h$, and $f(X_A + h)$ to plot point **B** by using *Plot As (x, y)* from graph menu;

- Construct the secant line AB and find it slope;
- Plot point ***P*** by selecting X_A and the *slope measurement* AB then choose *Plot As (x, y)* from Graph menu;
- Select point **A** and point **P** and choose **Locus** from Construct menu.

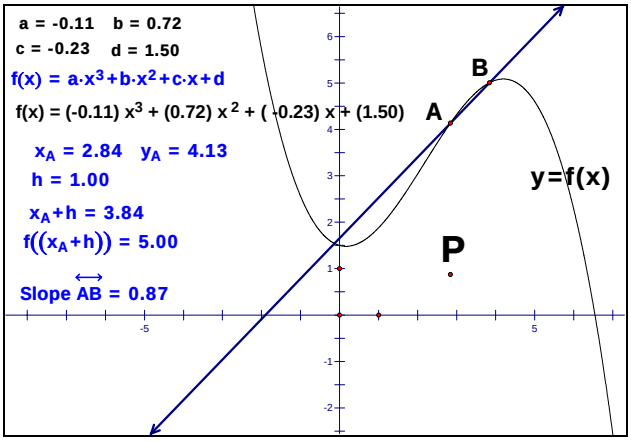

Figure 1: Secant Line

(2) The connection of algebra and calculus:

The students were told to guess and discuss what graph of the derivative of a function they should get before they used GSP to investigate graph of the derivative of a function at a point and slope of the tangent line. They had to explore the relationship of the locus of point P and graph of the derivative of a function when the value of the increment ***h*** was changed. The graph menu was used to plot the derivative of a function as follows:

- Select function $f(x) = ax^3 + bx^2 + cx + d$, then choose *Derivative* from Graph menu;
- Plot the derivative of a function by selecting $f'(x) = 3ax^2 + 2bx + c$ on the screen and choose *Plot function* from Graph menu.

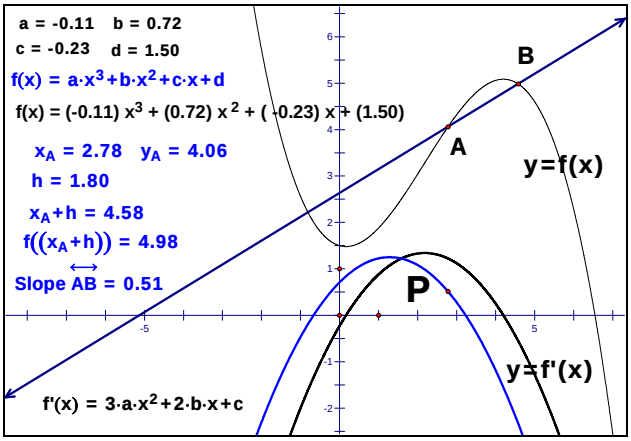

Figure 2: Graph of the Derivative of a Function

Students explored by changing parameter ***h*** or dragging the slider for ***h*** until it was closed to zero (0) and observed how it affects the locus of point P. Students worked in pair, observed, discussed and wrote their findings about:

- the behave of point A, point B, and slope of secant line AB; and
- the relationship of the locus of point P and graph of $f'(x) = 3ax^2 + 2bx + c$ (the *Derivative* of $f(x) = ax^3 + bx^2 + cx + d$).

Example 2: Calculus-mathematical problem solving:

> *"A man wants to design a box with no lid from a cardboard rectangular in shape size 6 feet x 8 feet. This box can be filled with maximum material. What is the dimension of this box?"*

This classic box problem is one of the examples of mathematical problem solving in calculus in upper secondary mathematics textbook in Thailand (IPST, 2001). In the textbook, the problem was described and emphasized more on the manipulation of symbols and notion of derivative.

Therefore, the teachers who were involved in the action research taught this problem by constructing representation in mathematics. The mathematics lesson started from cutting a paper rectangle in shape with size 6 inches x 8 inches, then cutting congruent square from the corners of this rectangular paper, and drawing a mathematics modeling. The next activity was the use of GSP to explore and solve the problem. The following activities describe how GSP can be used in connecting geometry algebra and calculus in solving this box problem.

Using GSP to sketch and explore the box problem

Start the Geometer's Sketchpad program and choose *New Sketch* from the File menu, then continue with the following activities:

(1) Construct a model of a box:

- Construct rectangle ABCD size 6 cm x 8 cm;
- Construct fixed segment ***EF*** with 3 cm and change to dash line; (Students discuss: Why 3 cm)
- Construct ***G*** as a point on segment EF, construct segment ***EG***;
- Construct 4 circles with radius EG and center at A, B, C and D;
- Construct intersections of the 4 circles and sides of a rectangle;
- Draw segments to construct a square at each corner and a rectangle to be a bottom of a box with no lid and hide circles;

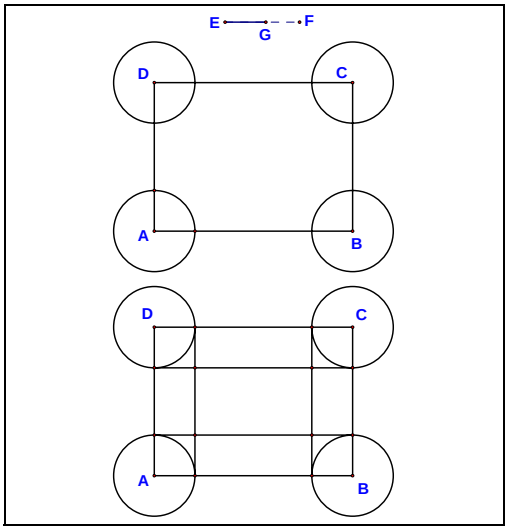

Figure 3: A bottom of a box

- Label the model of a box with no lid and construct polygon interior using construct menu. As shown in Figure 4 on your right.

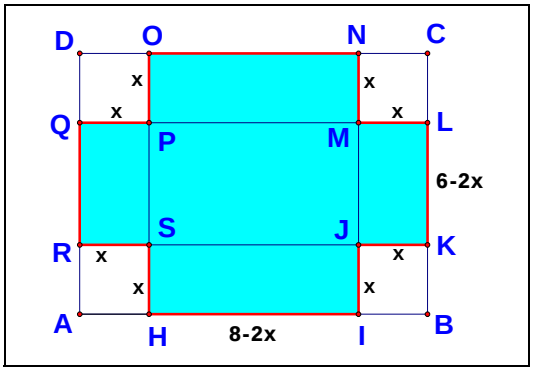

Figure 4: A model of a box

(2) Calculate the volume of a box:

- Measure length AH, HI and KL using Measure menu
- Select Measure menu and choose calculate then click at the measurement of AH, HI and KL on the screen
- Click OK

The value of the box volume will appear on the screen.

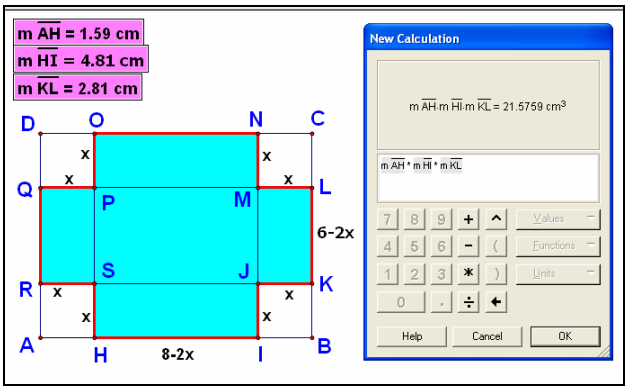

Figure 5: Calculate volume of a box

(3) The connection of geometry and algebra:

- Plot point of the measurements of the length ***AH*** and value of the box volume by selecting the measurements of the length of ***AH*** and the volume on the screen and choose **Plot As (x, y)** from Graph menu;
- Construct *Locus* of point ***T*** by selecting the plotted point *T* and point G, then choose **Locus** from Construct menu.

You should get a plotted point ***T*** and its locus as show on Figure 6.

- Animate point ***G*** by using Edit menu to observe the connection of geometry and algebra.

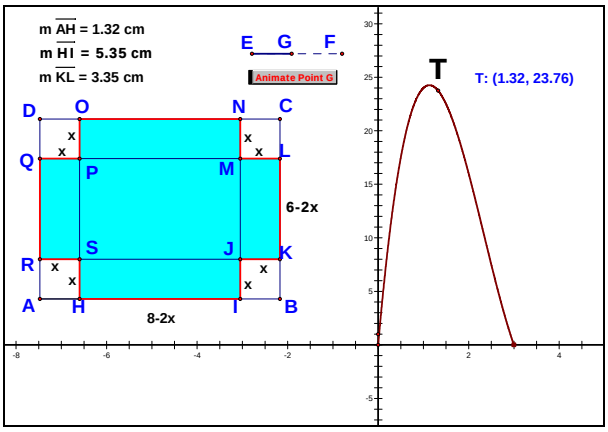

Figure 6: Connection of geometry and algebra using GSP

(4) Plot graph of function $f(x) = x(6-2x)(8-2x)$:

- Choose **New Function** from the *Graph menu*;
- Enter function $x(6-2x)(8-2x)$ in the *New Function* dialog box;
- Select f(x) = $x(6-2x)(8-2x)$ on the screen and choose Plot Function from Graph menu; Graph of

  f(x) = $x(6-2x)(8-2x)$

  appears as Figure 7.

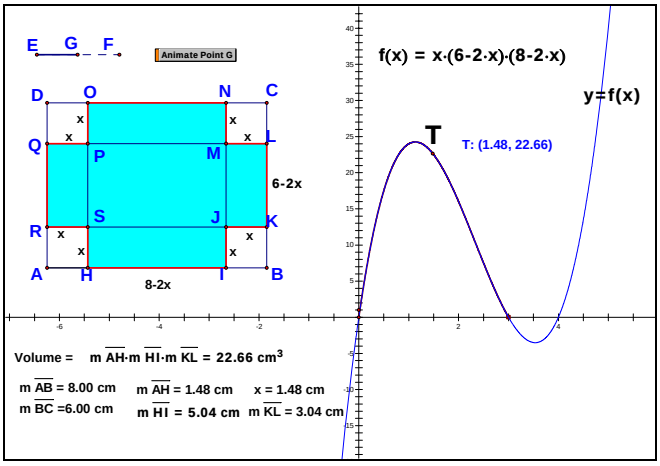

Figure 7: Graph of a function of the volume of a box

(5) *Plot graph of the derivative of a function f(x) =* $x(6-2x)(8-2x)$:

- Select f(x) = $x(6-2x)(8-2x)$ on the screen and select *Derivative* from Graph menu. The derivative of function $f'(x) = 12x^2 - 56x + 48$ appears on the screen;
- Plot graph of f '(x) = $12x^2 - 56x + 48$ using Plot Function from Graph menu and adjust the *Grid Form* to rectangular grid;
- Construct a perpendicular line from point T to x-axis as show in Figure 8.

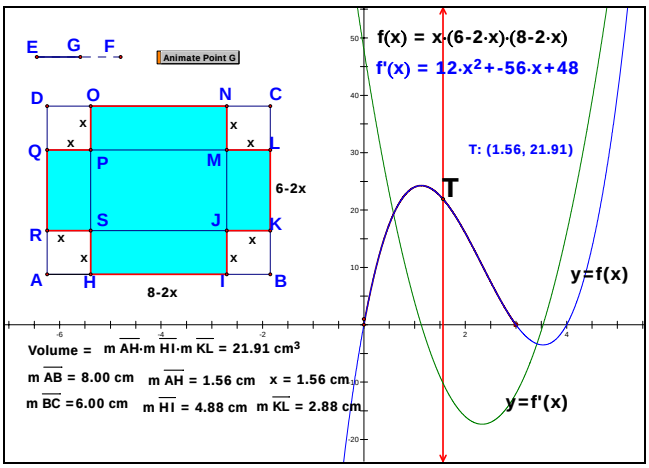

Figure 8: Connection of geometry algebra and calculus using GSP

Students worked in group discussed and explored by dragging point ***G***, or animate point ***G*** to explore the plotted point (T) move to correspond to different size of the length and the volume. Students discussed and explained the following:

- What the coordinates of the high point on the graph are and how they are related to the length of ***AH*** and the volume?
- Identify the domain of the function within the context of the problem situation.
- How is the perpendicular line from point T to x-axis related to the highest position of point T, x-axis, and graph of the derivative f '(x) = $12x^2 - 56x + 48$?
- How is the graph of the derivative f '(x) = $12x^2 - 56x + 48$ related to the highest position of point T?
- Find the dimension of the box which maximize the volume and then give the maximum volume.

Research Findings

As described earlier, this action research aimed to find out in how GSP can be used as a tool in mathematics classes and what are the effects of GSP toward students' attitudes in mathematics. The *Example 1* and *Example 2* in the previous section described how GSP had been used in the study. The summary of the findings from the action researches are described as follows:

1. *The use of GSP as a tool in mathematics and Thai mathematics teachers' perceptions*

The mathematics teachers who were involved in action research study revealed that GSP can be used as a tool in their mathematics class, especially in algebra and calculus. The teachers changed the way mathematics was taught. It brought mathematics into the technology world. The teachers found out that using GSP in their mathematics classroom had positive effects toward the teaching

and learning. The examples of the teachers' perception on the use of GSP as a tool in mathematics are as follows:

- It is good for slower students they can check their work, give immediate and individual confidence. The students investigated what the average rate of change between two points on a function looks likes and the slope of the secant line between two points. The students were able to explain that when *h* gets smaller one point approaches the other, average rate approaches instantaneous rate. These explanations were from the students while they worked in group discussion, these kinds of explanations did not happen before;
- The teachers have to teach concepts in mathematics together with the use of GSP. Students used GSP in computer lab and later on they have to work on paper-and –pencil again;
- Teachers have to design challenge questions for students to discuss rather than just finding the answer only;
- The teachers must have the knowledge not only how to use GSP but also how to correct all trouble shooting. Teachers must know when and how to teach mathematics using GSP;
- With the use of GSP, the students pay more attention in studying mathematics. The teachers reported that if they went to class not in time the students will look for them and encouraged them to teach mathematics with extra time. This kind of situation has not happened before; and
- The issues on the use of computer labs in the sample schools were:
    - There are few computer labs in my school, the lab reserved for teaching computer subject only. I am not able to teach mathematics in computer labs;
    - I did not get permission to teach mathematics in computer labs;
    - The teachers could not use GSP to teach mathematics in computer lab as much as they want to;
    - The students can use GSP in computer lab only lunch time or after class.

2. *Students' attitude toward mathematics*

Results from students' semi-structured interview revealed that the students now liked to learn mathematics because they were able to do many ways in solving mathematics problems. Teachers encouraged them to think and find their own ways to solve problems. The students revealed that when teachers explained mathematics by using GSP the graphs illustrating ideas were not only clear but also made the concept much more basic and easier to understand. The students liked the new way of teaching mathematics incorporate with GSP.

The examples of Thai students' perception on the use of GSP as a tool in mathematics are as follows:

- Now, the mathematics class is fun and interesting. GSP can make our learning in mathematics much more easier and interesting;
- I think using GSP is save a lot of time, faster and convenience;

- I think GSP has to use after the teacher taught the basic concepts, how to calculate *x* and *f(x)* value to get coordinate and plot graph;
- Calculus became easier and more interesting, now I understand why we have to set $\frac{dy}{dx} = 0$ in order to find the maximum value. It gave me a deeper and better understanding of the lessons because of the high-tech capabilities;
- The GSP gave me a clearer picture of how the graphs of the derivative f '(x) look like;
- I never use GSP before; it is quite difficult at the beginning. But now I like to draw model before I solve the mathematics problem, I can animate the point and it help me a lot.
- With GSP, we were able to visualize the characteristics of the graphs such as when the graph is increasing or decreasing;
- Using GSP is very useful, I learn better and faster. Mathematics has become more fun and easy to learn;
- The activities made us do a lot of discovery and exploration, which were impossible without GSP. This made us discover the characteristics of the graphs of different functions, especially the complicated ones;
- The activities that go with it enabled us to work by pairs & group. Learning mathematics became easier;
- The answer from GSP sometimes may not be accurate. But it is not a problem at least I can see the model and the graph.
- It is easier when you see the graph on the screen. It just like answer the problem without so much energy used, you must know your work and know what you are doing;
- Learning mathematics has become more enjoyable and I learned a lot from the dragging and explorations;
- Complicated problems can be solved in an interesting way and can be understood better.
- Much quicker than doing the graph manually. I found it helpful; I can check my graph just in case I made a mistake;
- GSP helps you spend time understanding the mathematical reasoning of what you are doing. Instead of spending time trying to work out long mathematical computation.

Conclusions

I had shown that the Geometer's Sketchpad is a very useful and effective tool for both teacher and students. GSP changes the way mathematics is taught, with GSP the teachers are able to connect geometry algebra and calculus in order to solve mathematical problem solving. It is clear that, GSP is an instrument in enabling them to obtain a deeper insight into the concepts. Best of all, most of the students have positive attitude toward mathematics after they learned mathematics using the GSP.

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