Flexible Solving Strategies in Algebra Remain in the Long Term

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Abstract: Reduced teaching hours of mathematics in primary and secondary schools in Japan seem to enhance surface learning among Japanese students. If a student surface-learns mathematics, he/she simply memorizes formulas and uses them without considering the embedded mechanisms. His/Her solving strategy depends on the formulas as black boxes and the usage of them according to the manual. It is quite rigid, and his/her answer sheet hardly includes any qualitative analysis, numerical confirmations, graphs, or explaining sentences. Flexible solving strategies and rich descriptions on answer sheets, we think, make a useful measure if the student has overcome the surface learning.

In this preliminary study, we tried to confirm if students' flexible solving strategies in algebraic calculation, which they learned with the use of our web-based instruction system two and a half years ago, still remain in their problem solving procedures in a fundamental engineering subject.

We selected a problem of a basic electrical circuit, where it takes time if a student sticks to formulas and symbolic analysis, but it is easier to answer if he/she uses graphs or qualitative analysis. Almost all the students could write the key formula for the problem in their answer sheets, but some students stopped there without adding any meaningful description. They apparently have rigid solving strategies, which must be related to the surface learning. However, the students, who have learned flexible solving strategies in algebra, have more flexibility in their problem solving strategies. We think it makes an evidence to support that flexible problem solving strategies in algebra remain in the long term.

1. Background

Deteriorated test scores of some Japanese university students in algebra were reported [1]. Okabe et al pointed the cause as reduced teaching hours of mathematics in primary and secondary schools along with the rejection of mathematics from the entrance examinations of those universities.

Fujisawa warned the invasion of surface learning among Japanese students [2]. Surface-learners memorize selected keywords or data blindly without considering the background information. They believe that it is an *efficient* way to prepare for the examinations in schools including mathematics.

We also have been observing several surface learners among fresh students to our college. They may enter the college as model students but fail in engineering subjects mostly because of their learning styles. Many of them could be detected if teachers observe their answer sheets closely because of their poor descriptions.

In algebraic calculations, surface learners make curious mistakes in their calculations [3]. Their erroneous calculating steps are guided by patterns similar to but not equal to mathematical rewriting rules. Because they have difficulties in separating those patterns from right rewriting rules by themselves, they cannot find errors in calculating steps done by others, too. They have low sensitivities to errors in algebraic calculations.

If a student surface-learns mathematics or engineering subjects, his/her solving strategies become quite rigid, and his/her answer sheet hardly includes any qualitative analysis, numerical confirmations, graphs, or explaining sentences. Rich descriptions on answer sheets make a useful measure to assess if the student has overcome surface learning.

In the former papers [4, 5], the authors had shown some examples of surface learning in Japanese students' algebraic fractional calculations, and also a remedial lessons to guide surface learners to more meaningful learning style using their web-based learning system (Figure 1).

The web-based learning system helped a student in;

1) showing a numerical counter example of the erroneous calculating step to let him/her recognize that he/she actually made a mistake,
2) showing a correct calculating step according to a mathematical rewriting rule responding to the request of the student one by one,
3) and providing the calculating opportunity where his/her every calculating step was evaluated immediately and got the feedback telling if the step was correct or not.

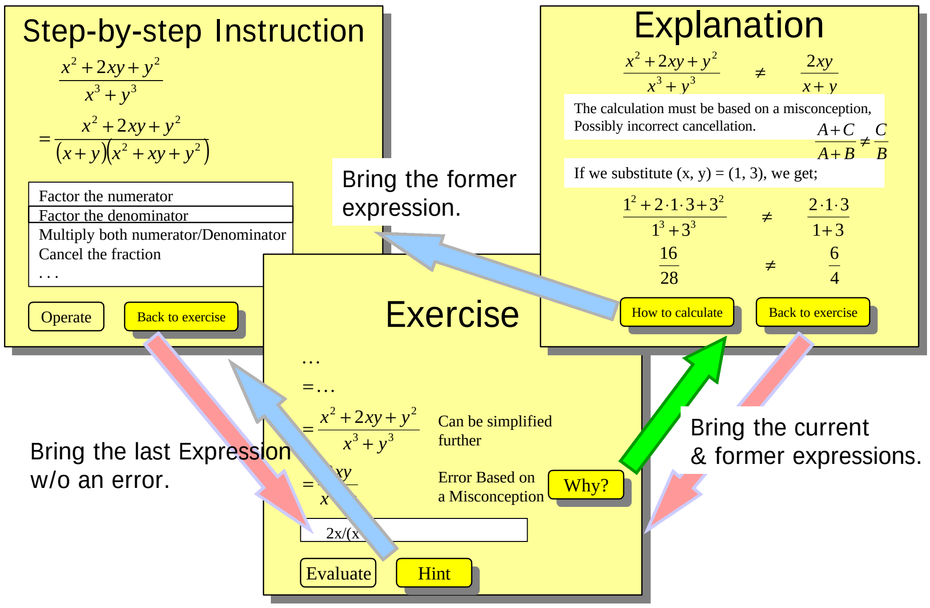

Figure 1 Three Pages of the Web-based Learning System of Algebraic Fractions [4, 5]

2. The Relation of This Study with Former Studies

In 2002/2003 school year, six students (students A in Table 1) in the first grade of our college (age 15) were selected for the remedial lessons of algebraic fractional calculations with our Web-based learning system [4, 5]. They were the lowest performers in the class (students A, Bs, and Bo in Table 1) at the pre-test of algebraic fractional calculations.

Table 1 List of activities and related students in this study

Date	Activity	Students		
		A	Bs	Bo
Oct. 2002	Pre-test (Calculating algebraic fractions)	6	36	
Nov. 2002	Remedial lessons with Web-based learning system	6	--	
Dec. 2002	Post-test 1 (Calculating algebraic fractions)	6	4	32
Feb. 2003	Post-test 2 (Finding errors)	6	4	--
Jun. 2005	Post-test 3 (qualitative analysis in an electrical circuit)	6	4	21*

* 11 students Bo are absent from our college on June 2005 because of their extra study abroad

After the remedial lessons, their performances were compared with their classmates (students Bs

and Bo in Table 1) in two types of post-tests: a calculating test and finding-errors test. Students A were as good as students B (Bs and Bo) in post-test 1 (calculation), and better than students Bs in post-test 2 (error-finding). Students Bs were the lowest performers among students B (Bs and Bo) in the pre-test, but better than students A in it.
In this study, solving strategies of students A are to be compared with the one of students B (Bs and Bo) in post-test 3 in Table 1, a test of qualitative analysis in an electrical circuit. Students A and Bs, fourth grade students in 2005/2006 school year, are the same students as former studies. Only 11 students of group Bo are missing in this study because of their extra study abroad.
At the error-finding post-test 2 in Table 1 [4, 5], a student was required to evaluate the printed answer sheet with four correct calculations, three calculations with careless mistakes, and two calculations with errors influenced by typical incorrect calculating patterns. Their task is to find the errors in whole calculating steps. When they found an error in any of the calculating steps, they were instructed to point the position and to describe what kind of error it was.
In the post-test 2, all the students (A and Bs) succeeded to identify the correct calculations but failed to find some of the incorrect calculations. While the students A identified 83% of careless mistakes and 83% of errors according to incorrect rewriting rules, the students Bs only identified only 50% and 63% respectively. The authors concluded the high sensitivity was one of the evidence that the students had escaped surface learning and gained more flexible solving strategies of algebra. They also expected that the flexible solving strategies would ease the students' learning of engineering subjects in their higher grades, and remain in the long term.

3. Engineering Problem Solved in this Study

In this paper, we studied if the students A still keep the advantage of flexible solving strategies in algebraic calculations to students Bs even after two and a half years of their remedial lessons. For that purpose, we prepared the post-test 3 (qualitative analysis in an engineering problem).

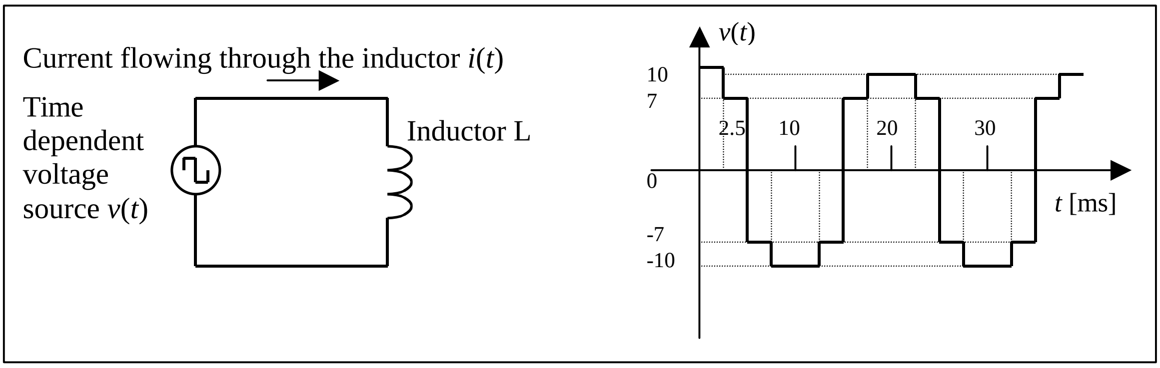

Figure 2 The electrical circuit and the applied voltage function $v(t)$

The problem requests the students to draw a graph of time dependent electrical current flowing through an inductor (coil) when a voltage applied across the inductor is given as a graph of the function (Figure 2). This is a fundamental problem of electrical circuit and the relation of the voltage $v(t)$ and the current $i(t)$ is well known as Faraday's law $v(t) = L\frac{di(t)}{dt}$, where L is the self inductance of the inductor in [H].
Because the voltage is given as a series of columns (constant function) of different values, it is easy

to figure out the graph of the current *i*(*t*) as a series of connected straight lines if one understands graphical influence of integration. Actually, qualitative analysis is enough to solve the problem. As we have expected, most of the students remembered the Faraday's formula and wrote it down on their answer sheets. Many students drew the graph of the current *i*(t) immediately afterwards although they were different in completion.

But some students stopped their answer just after writing the formula. There were no graph or no descriptive sentence. They either could not understand the graphical influence of integration because of their surface learning of calculus and/or fundamental electrical circuit theory. The rigid solving style, where without detailed procedures they could not start writing their solutions, is a typical feature of surface learning.

This problem becomes a little complicated when one sticks to symbolic solution because the voltage function in symbolic expression is divided into several constant functions. Integrating the functions with appropriate initial conditions repeatedly becomes a long procedure and takes some time, which prevents the surface-learners from starting the procedure. The problem is expected to measure the flexibility of the students solving strategies.

4. Skill of Qualitative Analysis in Engineering Problem

There are three types of answers in their answer sheets;

a) Mostly a graph

Because the students are requested to draw the graph of current *i*(*t*), many answer sheets include a graph of *i*(*t*). Some of the graphs show every feature of *i*(*t*), but the others show some of it.

b) Only symbolic calculation

Some answer sheets have no graph in them. The students apparently tried to deduce the current *i*(t) by integrating the function *v*(t), but *all* failed. Their solving strategies are quite rigid.

c) Both of them

Perfect answers but they are difficult to evaluate in this study. If the student has deduced the function *i*(t) first by integrating *v*(t) and draw the graph using the function, we cannot tell how flexible his/her solving strategies are.

Considering the three types of answers, we evaluated their answers according to the following three criteria;

1) If they draw graphs of *i*(*t*) in their answer sheet, add 1.0 point.
2) If the graphs show the phase delay as the result of integration, add 1.0 point.
   When the phase is shifted to the opposite direction, the point is reduced to 0.5.
3) If the graphs are a series of connected straight lines, add 1.0 point.
   For a curved line but similar to the straight lines, add 0.5 points instead.

We call the total points as "Completion of the Graph" in Figure 3 and 4. Completion of the graph for all the students B (Bs and Bo) disperse from zero to three points, and the average point is 1.84, which means the average student in group B can select to draw a graph, mostly knows the phase shift caused by integration, but cannot tell the function as connected straight lines in his/her qualitative analysis. Several answer sheets of highest score 3.0 have complete procedure of quantitative analysis. Almost all the answer sheets of lowest score 0.0 have no description than the Faraday's formula. The completion of the graph of students A (average: 1.92 points) is as high as

the one of students Bs and Bo.
Also, the two test scores, the sensitivity to errors in error-finding post-test 2 and the completion of the graph in post-test 3, are compared in Figure 4. They are correlated if we exclude two students A_4 and Bs_4. The correlation implies that flexible solving strategies developed in algebraic calculations remain in relatively high ratio of the students after two and a half years and appear as the skills of qualitative analysis in engineering problem solving.

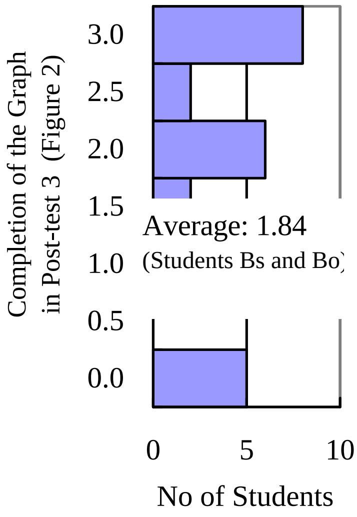

Figure 3 Completion of the graph for students B (Bs and Bo)

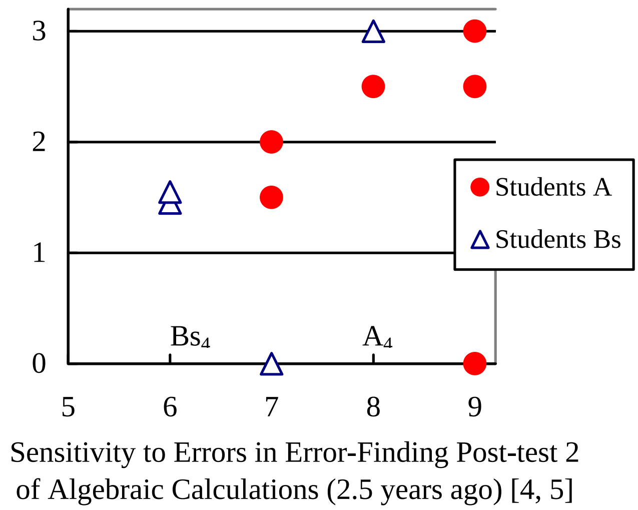

Figure 4 Correlation of sensitivities to errors and completion of the graph

5. Discussion

The students who understand the nature of integration and fundamental knowledge of electrical circuit are expected to analyze the circuit current qualitatively first, recognize the phase shift of the graph caused by the integration, and draw a connected lines for the function $i(t)$ in this problem, even though most of the other problems in this subject need quantitative analysis. In this case, we think the problem fits the purpose of measuring the students' skills of qualitative analysis as an evidence of flexible problem solving strategies. And the answer sheet with no graph, which gets zero points in the completion of the graph, has every feature of surface learning. Although it is rather difficult to judge the perfect answers that use both graphs and completed quantitative analysis (there are three answer sheets from students Bo), completion of graphs in the other answer sheets tells the students' skills of qualitative analysis.
We also have to remind the fact that there is no student who has low sensitivity in the error-finding post-test 2 of algebraic calculations in former studies but high completion of the graph in post-test 3 in this study (There is no dot in the upper-left corner of Figure 4). Although the high sensitivity to errors in algebraic calculations does not guarantee the success in mathematics and engineering subjects, we are afraid that rigid solving strategies we observed in algebraic calculations have fatal influence in learning those subjects.
We have confirmed the need of teaching flexible solving strategies *in algebra*, we think.

6. Conclusion

We have found a correlation between the sensitivity to errors in algebraic fractional calculations and the skills of qualitative analysis in fundamental electrical circuit, although the number of students is limited and there are a few exceptions. Both evaluations are designed to measure how far the students are from surface learners and how flexible their solving strategies are. The result suggests the long duration of the students' flexible or rigid solving strategies. We confirmed the importance of teaching flexible solving strategies in algebra before their engineering education, and the web-based learning system may become the help.

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