Prospective Elementary School Teachers' Use of Graphing Calculators to Solve Systems of Linear Equations and Inequalities

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Abstract: *This study involved three classes of prospective teachers. In a pilot study a class of prospective lower secondary school mathematics teachers participated in the development of an integrated unit on solving systems of linear equations and systems of linear inequalities using graphing calculators. The unit was then presented to two classes of pre-service elementary school teachers. Results showed that one of the two classes of prospective elementary school teachers performed at a comparable level to the pilot class of secondary school teachers. Findings also confirmed that both classes of future elementary school teachers performed on the calculator-based activities at a level that was consistent with their overall performances in their mathematics content classes. Analysis of the students' errors in solving the linear systems revealed that most of the non-computational errors were errors of omission or errors due to a lack of familiarity with properties of linear graphs or with how to interpret order relations graphically.*

Recent efforts in the U.S.A. to enhance pre-service programs for future elementary school teachers have often focused on using graphing technology to improve these students' expertise with mathematical systems. Increasingly, these systems have included both systems of linear equations and systems of linear inequalities ([11]). Finding solutions to these systems has proven to be a challenging task for students despite an ongoing emphasis in state and regional mathematics frameworks ([5], [8]).

Some popular mathematics content textbooks for prospective elementary school teachers include sections on solving systems of linear equations ([3]). However, the topic of solving systems of linear inequalities continues to be less frequently included in curricular materials for future elementary school teachers. Fortunately, technology has provided opportunities to connect the solutions of systems of both linear equations and linear inequalities on the increasingly accessible platforms provided by graphing calculators.

The study described in this report involved three classrooms of prospective teachers. In a pilot study a class of prospective middle school mathematics teachers participated in the development of an integrated unit on solving systems of linear equations and systems of linear inequalities using graphing calculators. The activities spanned two 80-minute class sessions and included both whole-class and individual work in solving the two kinds of linear systems. The pilot test results led to refinements in the activities and the pilot test results showed that the future middle school teachers' performance on the individual activities were comparable to their performances in both prior and subsequent assessments in the course.

Observations of the prospective middle school teachers' work on the pilot test activities involving systems of linear equations revealed that these students were familiar with using graphing calculators to graph and solve systems of equations. As a result, these activities appeared rather routine to them. The most common errors on the pilot test activities about systems of equations were (1) using terminating decimals to represent repeating decimals in solutions and (2) incorrectly applying the distributive property of multiplication over addition when rewriting fractional coefficients as integer coefficients in equations. One-third of the pilot test participants scored 100% on the activities on solving systems of linear equations; the mean of these scores was 89% while the median was 86%.

Observations of pilot test participants while they worked on activities involving systems of linear inequalities showed that they appeared to feel more challenged by these activities and they seemed to enjoy these activities more than the activities on systems of linear equations. Results from this portion of the pilot test revealed the need to rewrite one system to include the inequality $x \geq 0$. This was done on the blackboard for the pilot test participants while the activity sheets were annotated with this correction for subsequent administrations of the activities.

Pilot test participants also complained that it was difficult to position the trace cursor to boundary points or points of interest on the region formed by the graphs of the inequalities. This was a difficulty inherent in the calculator application that was used in this set of activities and the researchers encouraged the pilot test participants (and participants in subsequent administration of the activities) to experiment with the arrow keys that control the movement of the cursor (Figure 1).

Figure 1 Participants Using Calculator's Arrow Keys to Control Movement of Cursor

Despite this difficulty, two-thirds of pilot test participants scored 100% on the activities on solving systems of linear inequalities and the mean of these scores was 91%. All but one of the pilot test participants correctly reported the coordinates of the points of interest or boundary points of the regions formed by the graphs of the inequalities by using exact decimal or fractional representations.

Following the pilot study, four 100-minute sessions were used to present the activities on linear systems to two classes of pre-service elementary school teachers. During the first two sessions, students reviewed and refined their knowledge of algebraic solutions to systems of linear equalities and systems of linear inequalities. During the next two class sessions students participated in graphing calculator activities that emphasized tabular and graphical solution strategies. The third session was devoted to solving systems of liner equations; while the fourth focused on a calculator applications approach to solving systems of linear inequalities graphically.

The results of this study showed that one of the two classes of prospective elementary school teachers performed at a comparable level to the pilot class of future middle school teachers. Table 1 shows the average scores of the pilot test participants and the two class sections of future elementary school teachers on the activities involving systems of linear equations and systems of linear inequalities. Notably, the mean scores of the pilot group (91.39%) and class section 2 of the prospective elementary school teachers (91.96%) were comparable.

Table 1 Average Scores of Pilot Group and Class Sections of Pre-Service Elementary School Teachers on Activities on Systems of Linear Equations and Inequalities

	Systems of Linear Equations Activities	Systems of Linear Inequalities Activities
Pilot Group		
Mean Score	89.29	91.39
Median Score	85.71	100
Section 1		
Mean Score	73.81	76.67
Median Score	71.43	84.17
Section 2		
Mean Score	70.3	91.96
Median Score	71.43	95

Figures 2 and 3 display box plots of the participants’ scores for Sections 1 and 2. In both sections the plots indicate that the range of scores on the activities with the systems exceeded the range of the participants' course scores.

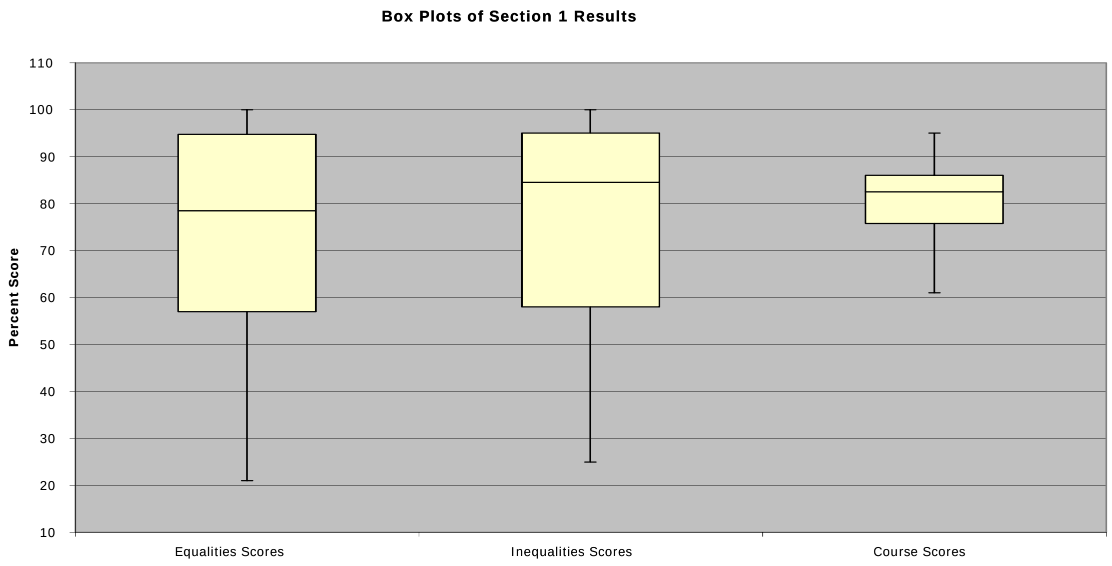

Figure 2 Box Plots of Section 1 Results on Activities and for the Course

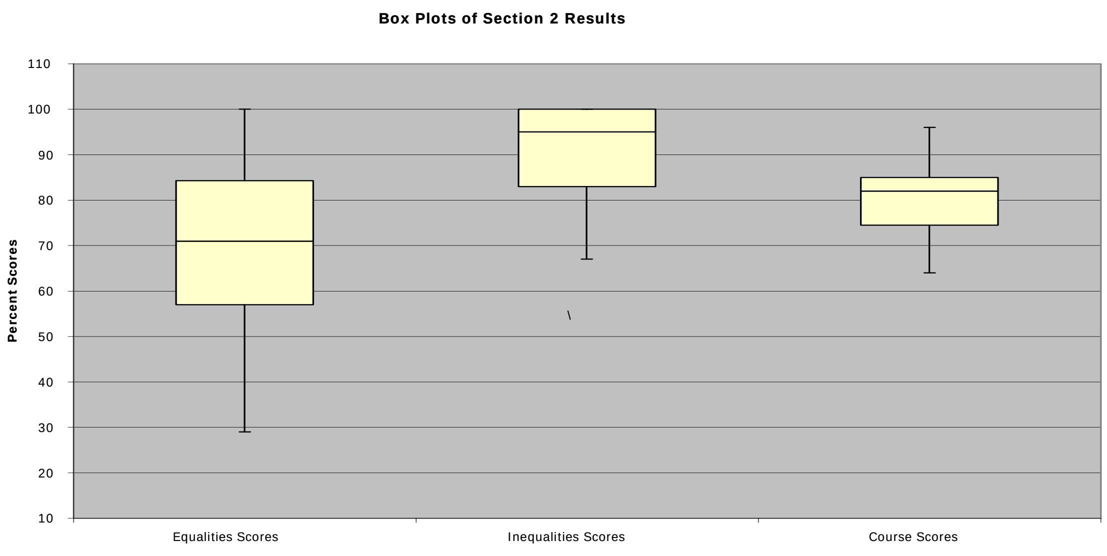

Figure 3 Box Plots of Section 2 Results on Activities and for the Course

Results also confirmed that both class sections of prospective elementary school teachers performed on the calculator-based activities at a level that was consistent with their overall performances in the course. Table 2 shows the mean scores of the two class sections of future elementary school teachers on the activities involving systems of linear equations and systems of linear inequalities and the mean of each class section's course scores.

Table 2 Mean Scores of Class Sections of Pre-Service Elementary School Teachers for the Activities on Systems of Linear Equations and Inequalities and for the Course

	Systems of Linear Equations Activities	Systems of Linear Inequalities Activities	Course Scores
Section 1	73.81	76.67	81.41
Section 2	70.3	91.96	80.78

Post-treatment qualitative data and videotapes of the prospective elementary teachers completing the graphing calculator activities showed improved confidence among the participants about their abilities to solve linear systems and increased confidence in their own mathematical expertise. Specifically, students were asked to respond to the following questions:
(1) Do you feel that the graphing calculator activities pertaining to the systems of linear equations/inequalities improved your understanding of the topic?
(2) Based on your previous experience with graphing calculators and your experience performing these activities, how do you believe that graphing calculators should be used in classrooms in school?
(3) Did you feel challenged by the graphing calculator activities pertaining to the systems of linear equations/inequalities? And
(4) What type of calculator do you own?
Responses revealed that most of the future teachers felt that the activities improved their understanding of the systems and most of the future teachers expressed an increased confidence in their abilities to solve similar systems by using graphing calculators. Results also showed that most of the pre-service teachers expected to use graphing calculators in their school classrooms and that a minority of the respondents owned a graphing calculator.

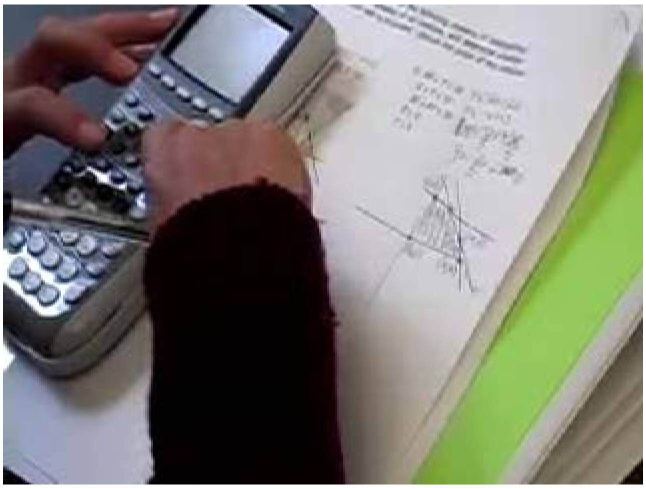

Figure 4 Image from the Video Study of Students' Performance on Activities Involving Systems of Inequalities

A video study of the students' errors in solving the systems of linear equations and inequalities (Figure 4) revealed that most errors were computational or errors of omission. The most common computational errors were incorrect evaluations of a linear function with an independent variable's value when attempting to find the value of the dependent variable. The most common errors of omission were missing coordinates of a vertex (or vertices) for the region representing the solution set of a system of inequalities. Other errors arose from a lack of familiarity with properties of linear graphs or with how to interpret order relations graphically. The most common of these errors stemmed from failing to recognize that linear functions with different slopes had intersecting graphs, incorrectly transforming a linear equation from the general form into the slope-intercept form, and incorrectly representing regions bounded by inequalities involving > or < as regions that included the boundary line(s).

While the average scores in Table 1 may indicate that participants made fewer errors in solving the systems of inequalities than in solving the systems of equations, there is no guarantee that any improvement in scores was due to an enhanced expertise with the systems of inequalities. An explanation for any reduction in errors appears just as likely to involve the use of the mathematically powerful calculator application in solving the systems of inequalities, or the increased experience in operating the calculators as a result of the practice from first solving systems of equations with this technology.

Without the technological boast provided by the graphing calculator application, the teaching and learning of how to solve systems of linear inequalities requires substantially more time and effort. As a result, mainstream mathematics texts for pre-service elementary school teachers often contain, at best, cursory descriptions about simultaneous linear equations in two unknowns ([9], [6]). However, lesson studies from Asia ([7]) and developmental studies from Europe ([10]) continue to accentuate the important role that technology and realistic situations play in developing tools for solving linear systems. Measures of just how important this role is in the teaching and learning the fundamentals of school mathematics is also increasing evident in State and local curriculum frameworks ([2], [1]).

Systems of linear equations and inequalities are a staple of beginning algebra that are subsequently used to develop linear programming concepts and the simplex algorithm for optimization problems. Consequently, the activities considered in this study could be extended by connecting the solutions of these systems to meaningful practical contexts thereby bringing scaled-down problem situations similar to those tackled by operations researchers into the classroom ([4]). Extensions such these could help prospective elementary school teachers gain a better understanding of the real-world problems that technology-aided mathematics can represent and solve so elegantly.

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