

# Problem Solving using Maple Animations

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## Abstract.

In our specialist mathematics degree, computer laboratory sessions with Maple are being used to fundamentally change the way we teach. Visualisation and animation are important issues with some modern applications of software. As an example, we mention Finite Element Method calculations of flow such as plastic filling a mould. Some modern FEM packages include a facility to animate the flow to assist with the visualisation of the flow process. Sophisticated computer algebra systems such as Maple (and Mathematica) have the facility to generate animations and this can be exploited in first year mathematics courses.

Since our students have already done some calculus at high school, we have run, since 1998, a new subject, nonlinear mathematics, that is taken concurrently with a fairly standard type of calculus subject. The nonlinear mathematics subject introduces some modern ideas, namely Phase Plane Methods and Iteration, and the use of Maple is integrated throughout this subject. Animation is introduced in this subject. The animations are mostly of parameterised families of curves and details are reported in "Animations using Maple in First Year" by Blyth [1].

Students enjoy the animation activities and so we have been encouraged to develop some more advanced animation materials which are being incorporated into our first year program. One topic which students have traditionally found difficult is the standard type of minimise or maximise problem of first year calculus course. To address this we have written our version of the Polya type of problem solving approach using Maple on the standard problem of a farmer fencing a plot of land with one boundary being a river. Students are asked (as an assignment) to use this approach to maximise the area in the Norman window problem.

Although not assessed, students were shown how to use animation of geometrical figures (generated with the Maple plottools package). The example used was a rectangle inscribed in a triangle, where animation was used to find the dimensions of the rectangle with the greatest area. To achieve this, a dynamical display of the dimensions and area of the rectangle is required. It is also instructive to superimpose an animated area versus rectangle width plot on the animated diagram. The maximum is very clearly illustrated and found with this approach.

Exercises (not for credit) are provided. These are to maximise the area of a trapezium inscribed in a semi-circle and the Norman window problem. These animations are very striking and are strong visually. However it is a lot easier to just use calculus, but not visually appealing. We demonstrate both and encourage students to do both.

*Mathematics Subject Classification (2000): 97C80, 97U70*

## 1. Introduction

The software packages Maple and Mathematica have been used in teaching and research in our department for about a decade [1-5]. Due to the success (from the perspective of both staff and students) of our Maple and Mathematica laboratory sessions, we have progressively increased the usage of these in our teaching and are fundamentally changing the way we teach. Since 1998, our mathematics program has included computer laboratory sessions for almost every course. This is usually work with Maple (we now have a site license for Maple). Teaching students of other departments, particularly from engineering and computer science, is a core activity of our mathematics department. In consultation with these departments, we have also increased the use of various software packages as appropriate to the course. Where the demand is for specialist packages (such as for Matlab toolboxes or commercial software Finite Element Method packages ...) they are used, otherwise Maple is incorporated in the course. This usage of Maple varies a lot from a little "tutorial" support, to major assignment work, to Web delivered material, to complete integration in the course with 100% of the assessment in an examination in the computer laboratory.

This paper discusses the introduction of some Maple lectures and assignments with programming of numerical integration, problem solving and problem solving with animations in a subject, MA910, that is offered in the first semester of first year. This subject is primarily the first semester of a traditional calculus course. MA910 is closely related to the course taken by the engineering students, but one of the topics studied by the engineering students at the end of the semester has been replaced by a Maple topic. The students who are enrolled in MA910 are in two groups, the first of which is studying a multi-major degree in science and will probably not take a major in mathematics. This student group has not already used Maple, so they have been provided with a separate (and more introductory) program which is not discussed here. The second group consists of students studying our specialist mathematics degree or our Dual Award of our mathematics degree and a diploma in information technology. This second group of students takes the nonlinear mathematics subject [1,2] and have been using Maple in two one-hour laboratory sessions per week. The work done with this second group of students, already experienced with using Maple, is described in the following.

## 2. Nonlinear mathematics and animation

Since the students in our specialist degree programs have already done some calculus at high school, we have run, since 1998, a new subject, nonlinear mathematics [1,2], that is taken concurrently with a fairly standard type of calculus subject, MA910. One of the objectives of the nonlinear mathematics subject is to create interest and to introduce some modern ideas, namely Phase Plane Methods and Iteration, with the use of Maple integrated throughout this subject. Animation is introduced in this subject. The animations are mostly of parameterised families of curves and details are reported in "Animations using Maple in First Year" by Blyth [1].

The starting point for animation is the Newton Method Animation worksheet which all of the students work through in a supervised laboratory session. A sequence of the family of curves  $y = 2x^2 - \sqrt{x} + c$  is animated in order to assist with the understanding of what was observed in a previous laboratory session using Newton's method. The animation parameter is  $c$  and the

animation can clearly illustrate why different values of  $c$  can give quite different behaviour with Newton's method.

An animation assignment asks students to form groups and select one of five problems to produce an animation and demonstrate the animation in the laboratory (for assessment). This is enjoyed by the students and a few even do extra animations just for fun!

Animation is also used in lectures to demonstrate the graphical interpretation of Newton's method and also to illustrate fixed point iteration with animated iteration diagrams for various possibilities such as where there are attractors, repellers, limit cycles and chaotic iteration.

### 3. MA910 and the numerical integration assignment

For the group of students in MA910 who were nearly finished the nonlinear mathematics course, three lectures and three supervised laboratory classes over three weeks were allocated to some Maple activities. This group had already used Maple extensively in topics such as Newton's method, animation (see comments above), fixed point iteration and solution of differential equations. In the MA910 course, topics covered include an introduction to numerical integration using trapezoidal rule and Simpson's rule. One lecture was devoted to a careful "revision" of trapezoidal rule and a discussion of how the algorithm is rewritten for efficient coding. This included some time showing (with a live computer projection) the details of the code in the Maple worksheet that was developed for this purpose. A hardcopy of the worksheet, with all output removed, was given to each student. Students were assisted with working through this worksheet in the supervised laboratory (which we both attended along with about 50 of the 60 students enrolled for this group). Students were then asked to complete the MA910 Maple Assignment 1, reproduced below (with the original Maple section formatting removed):

#### MA910 Maple Assignment 1

for the mathematics Degree and Dual Award group

#### The Assignment:

In a group of size 2 plus or minus 1, prepare a worksheet which has

a title, authors (name and student number), brief explanation and write-up for the following.

Hand in to Prof Blyth (in person or via the pigeon hole in the Maths Dept.) by 5pm Friday May 18, 2001.

#### Preliminary

Code the procedure `trap:=proc(a,b,n)` from the `trap.mws` worksheet AND **provide some CHECK** that it is running correctly.

Calculate  $p$  for  $f(x) = e^x$ ,  $a = 0$ ,  $b = 1$  where  $\text{error} = \text{constant} * n^{(-p)}$

#### Question 1

Define your group number,  $gpN$ , as the average value of the last digit of the student numbers (say  $gpN = (8+5)/2$ ) and define

$$f := x \rightarrow \sqrt{x^2 + gpN}, \quad a = 0, \quad b = gpN.$$

Consider the integral  $\int_a^b f(x) dx$  and the trapezoidal approximation for this integral.

1. Use a log log plot of trapezoidal method error versus  $n$  to show that the error follows a power law.
  2. Calculate the  $p$  here to check that  $p$  is (approximately) the same as before (in the preliminary question).
- Hint: remember to change the definition of exact for this new problem.

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### Question 2

Redo Question 1 after editing the plot command so that the  $n$  are doubled each time, from  $n = 2, 4, 8, \dots, 1024$ .

### Question 3

1. Edit the `trap()` procedure to calculate the Simpson method approximation (and CHECK!!).
  2. Use a log log plot of Simpson method error versus  $n$  to show that the error follows a power law.
- Hint: only use  $n$  between 2 and 32.
2. Calculate the  $p$  here.

We have a lot of direct contact with the students, but we also made careful notes about student performance on this (and the next) assignment. Most students were satisfactory, although we noted some difficulty with a lack of understanding of power laws, log log plots and the relationship between them. This has been covered at secondary school, but has not been understood. As a result there was some confusion about what was required and just what the power  $p$  is about. We will provide some extra work on this next year.

The other difficulty was the programming aspect of rewriting the code for Simpson's rule. Even though these students had been reproducing and using code for a variety of simple programs, they needed more help from us than we anticipated to edit the trapezoidal code to produce the Simpson rule code. We will address this next year with extra "hints".

## 4. The problem solving assignment

The second Maple topic was one which students have traditionally found difficult. It is the standard type of minimise or maximise problem of first year calculus course. This is covered in secondary school and (briefly) in first year in MA910. From our perspective, all of these problems start to look

the same, but this is not the case for the students who struggle with proceeding from a problem given in words, to formulate the problem mathematically and then solve the problem. To address this we have written our version of the Polya type of problem solving approach using Maple on the standard problem of a farmer fencing a plot of land with one boundary being a river. The max&min.mws worksheet developed for this is given in outline below where only the section headings (except for the preamble) are given.

## **Maximum & Minimum: Applications**

### **General preamble**

Many applications require us to find the maximum (or minimum) of some quantity. We will illustrate this with a simple example that is historically one of the first applications of the theory to a practical problem.

It is important NOT to get too focussed on all the computational details: the primary objective here is to understand the structure of the problem and the solution process. Thus each step of the (general) solution process is performed in a separate section below.

### **The problem: fencing some land alongside a river**

#### **Step 0: read the problem**

#### **Step 1: draw a diagram**

#### **Step 2: write the problem/information mathematically**

#### **What is to be maximised ...**

#### **What is the range for the one variable?**

#### **Find the max**

#### **Check**

#### **The solution is ...**

#### **Update the diagram ?**

The concepts and the Maple worksheet were treated in class. The students were (as usual) given a hardcopy of the worksheet, with all output removed, and asked to work through it (in the supervised laboratory class) and complete an assignment. The MA910 Maple Assignment 2 was to use this approach to maximise the area in the well known Norman window problem:

### **The problem: a Norman window**

A Norman window is constructed from a rectangular sheet of glass with a semi-circular sheet of glass fitted on top (of the rectangular piece). The amount of light that passes through the window is proportional to the area of the glass. What are the dimensions of the Norman window that has the maximum amount of light transmitted if it has a perimeter of 10m.

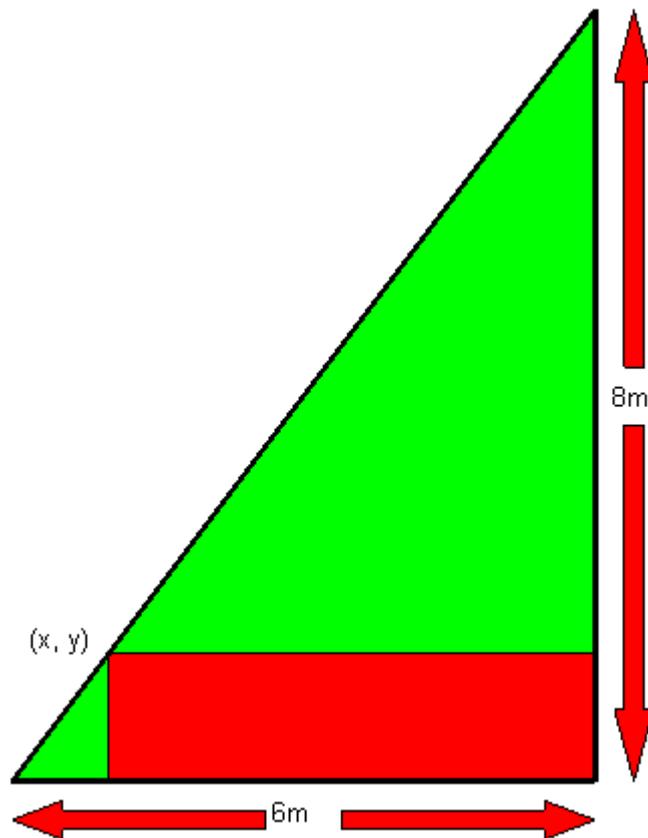
It was pleasing that most students did well on this assignment. They were familiar with the Maple commands and simple graphics requested (for the diagram) and they clearly understood what was required. We rated this activity and the student performance as very successful. The only negatives were the usually poor quality of comments and explanation. We strongly and repeatedly emphasize the need to provide a write-up: explanatory comments must be used and are required in order to gain full marks. For example, very few assignments had a comment to connect the area being maximised and the maximum amount of light transmitted.

## 5. Problem solving and animation

The diagram used in the provided Maple worksheet for problem solving was a simple construction using plot commands and textplot commands from the plots package. The assignment only required simple diagrams (but with appropriate labeling). We decided to introduce some of the commands available in the plottools package which provide the tools to construct more interesting diagrams. The diagram (with arrows) for the rectangle in a rectangle problem (see below) was built up with full expansion.

### Example 1: rectangle in a triangle

A rectangle is to be inscribed in a right triangle having sides of length 6 m and 8 m as shown in the figure.



- i) Redraw the diagram for any inscribed rectangle.
- ii) Use animation to display different inscribed rectangles with their corresponding areas indicated. Hence find the dimensions of the rectangle with the greatest area. Justify your answer by calculation (using calculus).

On a colour monitor, the triangle boundary is black, the rectangle and the arrows are red (dark gray here) and the rest of the interior of the triangle is green (light gray here). This problem and its solution was presented as a demonstration to students in class. The new element was the construction of the geometrical plots and the idea of using animation to find the maximum. The dynamic labeling of the plots in the animation was not a new idea since the students had already completed assignment work on this in the nonlinear mathematics course. The dynamic labeling of the dimensions of the rectangle and the area allows a very clear visualization. The ability to step

though the animation frame by frame while monitoring the corresponding dimensions and area allows for the solution of the problem from this animation.

It is also instructive to superimpose an animated area versus rectangle width plot on the animated diagram, see Figure 1. The maximum is very clearly illustrated and found with this approach where the animation is run to develop a feel for the problem and then stepped through frame by frame to determine the solution. The visualization of the geometry, the dimensions and the dynamic trajectory of the area graph is striking.

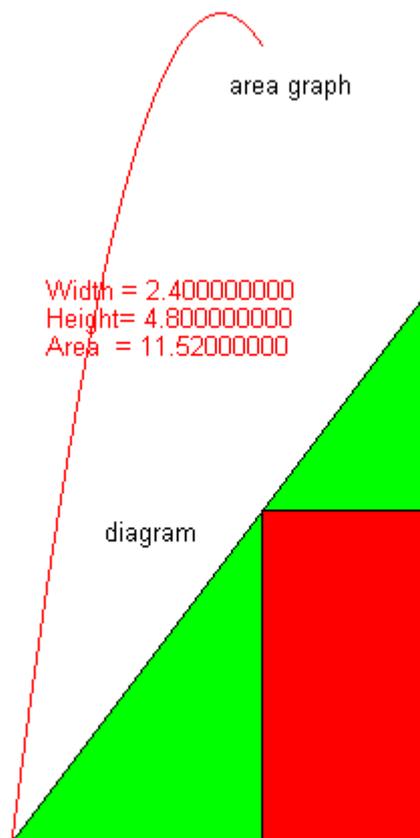


Figure 1. One of the frames in the problem solving animation.

Exercises (not for credit) are provided. The first is similar to the example demonstrated, but requires the maximum area of a trapezium inscribed in a semi-circle so that one side is along the diameter.

Since this work was presented as a classroom demonstration, the opportunity was taken to construct an overly elaborate Norman window diagram to illustrate the capability of Maple's plotting of geometrical objects. The diagram of Figure 2 was presented and the second (non credit) assignment was to follow a detailed list of hints to draw the diagram.

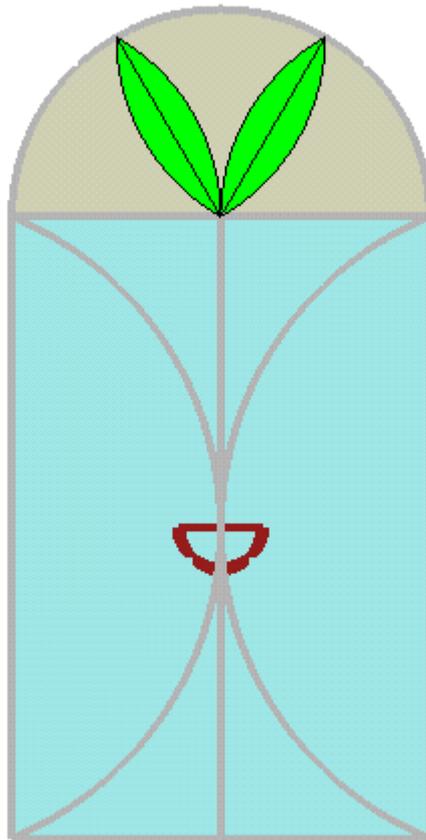


Figure 2. An elaborate diagram of a Norman window.

## Conclusion

The trapezoidal method topic was handled satisfactorily. Diagrams were used to illustrate the idea and plots of error were used to ensure that visual elements were utilized to assist with understanding. The students struggled with some aspects of the computation of the power in the power law and with how to rewrite the algorithm from trapezoidal rule to Simpson rule. For next year we will rewrite this worksheet and its assignment to include more support for these items.

The problem solving using Maple was very successful. The use of animations in problem solving is very striking and strong visually. However it is a lot easier to just use calculus for problem solving but not visually appealing. We demonstrate both and encourage students to do both.

There are costs associated with using and integrating packages such as Maple into teaching courses. Besides the hardware and software costs, the development time required is much higher than that for a traditional lecture course. As a guide we'd suggest that the development of a worksheet for a one hour laboratory session, using Maple or Mathematica, typically takes about 10 hours. In some cases, considerably more time is required: the problem solving and animation worksheet (discussed above) has taken in excess of 30 hours to develop. We have also made the four Maple worksheets for the trapezoidal method, its assignment, the problem solving and its assignment (the Norman

window problem) available for download from the subject homepage. The subject MA910 will be available online next year – the work required to implement that is another story to be told by the online developers Ian Grundy and Gary Fitz-Gerald.

The work needed to undertake course renewal with innovative integration of software and online delivery is difficult without the kind of support that we enjoy in our department. For the work reported here, we particularly wish to thank Gary Fitz-Gerald, Ian Grundy and Julie Saunders for their help and encouragement.

## References

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