Remedial Education of Symbolic Fractional Calculations Focused on Each Calculating Step

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The remedial education of symbolic fractional calculations using a WWW-based step-by-step instruction system is described in this paper.

The manual calculation skills of Japanese students have been gradually declining over the past decade, at least at the institutions some of the authors are working for. Their calculations have less accuracy and poor consistency. For example, although they can calculate numerical fractions, the same students struggle to determine the very first step of the addition or multiplication of symbolic fractions. What they lack, in our observation, is to think about the reasoning of each calculating step and to externally describe it.

Repeated Exercising does not improve the situation of some students. It seems to us that they have a tendency to repeat the inappropriate calculations induced from their own bug-rules in the exercises even after the inappropriateness is pointed out. They admit their mistakes but do not seem to reconsider their methods that produced the mistakes.

To compensate for their weakness and to provide them with the opportunity to think of the reasoning, we've introduced a WWW-based step-by-step instruction system into the remedial education for slow learners. The system requests the students to select one of the operations in a list on their computer display for forwarding a step of the calculation, and calculates only one step instead of the student.

Students were initially embarrassed since they were forced to think of the reason for the calculation. It took a long time to accomplish a very simple calculation in a step-by-step manner. However, as they got used to the method, they seemed to recognize the method the system guided them to do. After a series of trials using the system, their calculation skills were remarkably improved.

1. Background

In some Japanese higher educational institutions, the students' skills of manual calculations have been reported to be recently declining (Okabe 2001). Some teachers report that it is quite difficult to continue their classes without the help of remedial courses in high school mathematics. Without the remedial courses, the teachers have to stop some classes and start explaining the mathematical operations used to describe the theories in the middle of the classes. This phenomenon is predicted to

extend into the next decade because of the following two reasons. One of them is that the entrance examinations that used to maintain the minimum standard of high school mathematics are loosing their influence because of the decreasing number of young people relative to the capacity of the higher educational institutions. The other is that the contents of arithmetic and mathematics in primary and junior high schools will be reduced about 30% in the next standard curriculum.

Nishizawa and Yoshioka reported a countermeasure to stop their trend in the department of electrical and electronic engineering at their college (Nishizawa 1999, Yoshioka 2001). They had added a series of individualized pencil-and-paper tests as an auxiliary activity related to their algebra class. In every lesson, they gave their students small tests composed of fundamental problems that the students had already learned. If some of the students failed to answer the test successfully, i.e., the correct answers were less than 60%; the students were requested to have supplemental tests after the class until they succeeded in answer them. Since the problems of the supplemental tests were similar to the original ones and generally simpler than them, the tests expected 80% or more correct answers. The supplemental tests improved the knowledge and skills of the average students in their class.

However, the supplemental tests had less effect on two groups of students, the advanced students who did not need to attend the supplemental tests, and the least advanced students who could not make progress just by repeatedly solving problems. It seemed that they had tended to repeat inappropriate calculations induced from their own bug-rules in the supplemental tests. Thus we thought that the latter group needed more instructions before the exercises.

For the remedial education of the least advanced students, Yoshioka proposed a WWW-based instruction system (Yoshioka 2000). The system showed the calculating steps of symbolic fractions one by one using the principle proposed by Fuchs and Dominik (Fuchs 1999, Fuchs 2000). It utilized a Computer Algebra System in the way Buchberger suggested (Buchberger 1990).

In this paper, we show the actual method and the results of the remedial education using the WWW-based step-by-step instruction system.

2. Method

2.1 Paper tests

On May 15th, 2001, we conducted a paper-test of calculating symbolic fractions (Table 1).

Table 1 Example Problems given at the original test at May 15th.

$\frac{2x^2-2x+1}{x-1}-\frac{2x^2+4x-1}{x+2}$ $\frac{1}{x+1}-\frac{1}{x-1}+\frac{2x}{x^2-1}$ $\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)}$ $\frac{x+1}{1-\frac{1}{x+2}}+\frac{x+3}{1+\frac{1}{x+2}}$

First grade students (age 15) of the electrical and electronic engineering department took the test. Because there were no students who correctly answered more than five problems out of ten problems, all 42 students had to attend the following supplemental tests.

We prepared six test-sheets, each composed of five to seven problems, as additional tests. The test-sheets were graded on six levels according to their difficulty. The problems in the first level were the most fundamental ones, and the ones in the sixth level were the same or even to the problems of the original test. Her/his score at the original test determined the number of test-sheets for the supplemental tests. For example, if a student got three points on the original test, s/he did not have to answer the test sheets from first to third levels but had to answer the sheets from the fourth to sixth levels.

We scheduled ten supplemental-test sessions each composed of sixty minutes from May 22nd to July 3rd, and five sessions each composed of ninety minutes from July 10th to July 16th. In each session, every student was allowed to have two test-sheets at most in the former period, and three test-sheets in the latter period. The student who wanted to have an extra test had to make an appointment for the test-sheets beforehand through the WWW-based communication system (Nishizawa 1999). The system provided the student with the updated list of supplemental test-sheets s/he had to pass.

2.2 Remedial course using the WWW-based step-by-step instruction system

2.2.1 The overview of the WWW-based step-by-step instruction system

In the Web page for each student, a fractional expression as the problem to be calculated was displayed with a list of basic operations such as factorize/expand the numerator/denominator, cancel a fraction, and multiply both the numerator and denominator by the same expression, etc. Figure 1 shows an example of the Web page for a student on the computer. After every time the student selects an operation from the list on the screen, the system moves forward one step in the calculation according to the operation, then it adds a new expression in the page. By comparing the two expressions before and after the operation, the students were expected to learn the name and function of the operation, and hopefully it's meaning.

Figure 2 shows the process on the server computer during an instruction. When a student selects the operation on his/her instruction page, a list of mathematical expressions on his/her page and selected operation are sent to the WWW server with options and parameters (Step 1 of Fig.2). They are passed to MATHEMATICATM through the CGI program running on the server (Step 2 & 3). MATHEMATICATM rewrites the last expression into a new one by applying the selected operation, analyzes the new expression for commenting, and creates a two-dimensional graphical image of the expression into a GIF file (Step 4). All the methods are described as a custom function written in the MATHEMATICATM language. New list of expressions with comments and some characteristic data related to the new expression is sent to the Plug-In (Step 5 & 6). The Plug-In constructs a HTML file using the data and the names of GIF files, and sends it to the student's WWW browser

(Step 7 & 8). Since all these processes occurs on the server and the data sent to students are written in HTML, it is compatible with many WWW browsers including older versions running on slower machines.

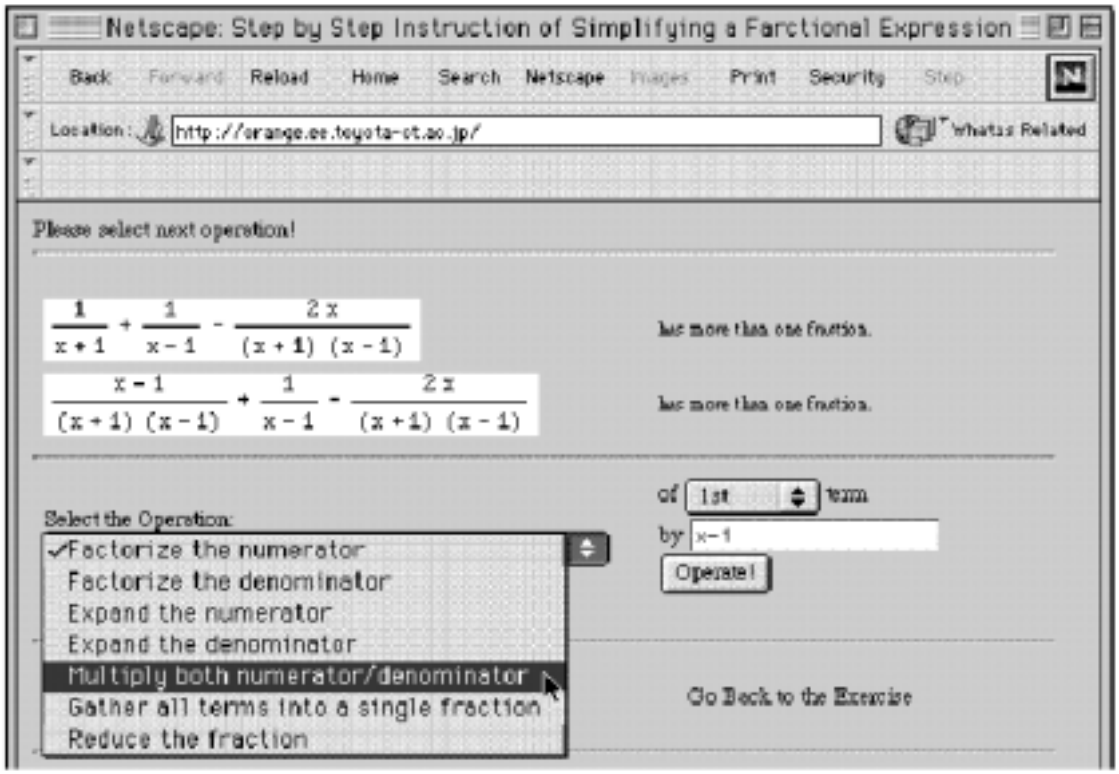

Figure 1 Example of the screen for the WWW-based step-by-step instructions

Figure 2 Process on the server during an instruction

2.2.2 Protocol for remedial course using the system

The remedial course using the WWW-based step-by-step instruction system was conducted in parallel with the supplemental-test sessions during the summer vacation from July 10th to July 16th. The duration of each session was 60 minutes. On the first day, a teaching assistant explained how to use the system and the students learned how to use it. On the following four days, the students used the system to determine the calculations in a step-by-step manner by themselves in all the available time (60 minutes). The teaching assistant stayed with them in order to help them use the system and answer their questions. The students attending the remedial course in the afternoon were able to attend the supplemental-test sessions in the morning. In this way, they had the opportunity to apply the knowledge learned in a remedial lesson to the supplemental tests on the following days.

In this report, we reviewed the answers written in the test-sheets and the learning records of the students stored in the system. We tried to measure the effectiveness of the remedial course by the change of their answers with time, how they developed the calculations with the help of the system, and the questionnaire of the students at the end of the course.

3. Results

3.1 Pencil-and-paper based additional exercises

Although all the students could not pass the original test conducted on May 15th, the results of the supplemental tests differed from student to student. We categorized the students into three groups according to the cumulative points of the supplemental tests on July 4th (Table 2).

Table 2 Students' performances for the additional exercises.

	Number of Students	Average Point on July 4th*1)	May 22nd – July 10th		July 11th – July 16th*4)	
			Sheets / Person*2)	Success Ratio*3)	Sheets / Person*2)	Success Ratio*3)
Group A	18	6.0	6.2	75%	0.0	-
Group B_1	9	4.6	7.9	51%	0.0	-
Group B_2	7	4.4	7.8	50%	1.4	46%
Group C	8	1.3	4.3	27%	2.9	61%

*1) If a student answers the all test-sheets successfully, s/he gets the maximum points: 6.

*2) The average number of test-sheets the students took during the period.

*3) The ratio of successful answer sheets over all the applied sheets during the period.

*4) After the start of the remedial course, which the students of Group C attended.

The students of Group A were the fast learners. They had 1.2 points on average at the beginning of the supplemental tests, but they finished all the test-sheets successfully by July 3rd by answering 6.2 test-sheets.

The students of Group B (B_1 & B_2) were the average students. They answered 7.9 test-sheets per person by July 10th, and succeeded in half of them. They left 1.5 test-sheets unsolved. They were divided into two subgroups, B_2 for the students who attended the supplemental tests after July 11th, and B_1 for the others.

The students of Group C were the slow learners. They answered 4.3 test-sheets, which were the lowest among all the groups, and succeeded in 27% of the tests, which was almost half that of the average students. Reviewing their test-sheets, we found not only calculating errors but also calculating steps apparently lead by bug-rules or misconceptions. They lacked some knowledge of calculating symbolic fractions.

A personal interview with one of the students in Group C revealed that he did not forward the calculating steps according to any mathematical rules. He rather converted an expression by applying a pattern that looked the same as a mathematical rule without considering the condition or the validity. For example, he explained his canceling of a fraction as "deleting the same numbers or symbols from the numerator and the denominator." His rewriting like $\frac{2x+3}{x^2+2x+1} \Rightarrow \frac{3}{x^2+1}$ was so natural that he could not recognize the inappropriateness, which he noticed after referring to a similar inappropriate rewriting $\frac{4+2}{3+2} \Rightarrow \frac{4}{3}$. What he lacked was the attitude to think of the reasons for each calculating step.

3.2 Remedial course using the WWW-based step-by-step instruction system

All the students in Group C attended the five-day remedial course using the WWW-based step-by-step instruction system. On the first day, their operations took time and they were inconsistent. Some of them had difficulty in selecting an operation from the list to forward the actual calculating step. In the following days, as they got used to the method, their operations got faster and more consistent. During the five sessions, a student completed 500 calculating steps on her/his screen on average.

The students continued to attend the supplemental tests held parallel with the remedial course. On the supplemental tests, the ratio of successful test-sheets for Group C jumped from 27% to 61% as shown in Table 2. As the ratio of successful sheets for Group B_2 stayed the same, the remedial course provided a good basis to explain the change in Group C.

Table 3 shows the results of the questionnaire done just after the remedial course. Most of the students, who had made many mistakes in the calculation of symbolic fractions before the remedial course, increased their confidence after the remedial course. Although four students evaluated the

supplemental tests based on pencil-and-paper as more effective than the step-by-step instruction based on the computer system, two students evaluated the step-by-step instruction higher than the supplemental tests. All said that the system was helpful (extremely: 1, relatively: 7) for them, and they judged the optimum duration for using the system as 34% longer than they actually did.

Table 3 The students' impression of the step-by-step instruction system (8 students)

Before the remedial course	I had no confidence in the calculation of symbolic fractions.	2
	I made many mistakes during the calculations.	6
	I had confidence in the calculations.	0
After the remedial course	I still have no confidence in the calculation of symbolic fractions.	0
	I do not make mistakes other than careless mistakes.	3
	I have confidence in the calculations.	1
I began to understand how to forward the calculation of symbolic fractions.		3
The system helped me to understand the calculating methods.		3
The supplemental tests based on pencil-and-paper are more effective than the step-by-step instruction based on the computer system.		4
The step-by-step instruction is more effective than the supplemental tests.		2

4. Discussion

The WWW-based step-by-step instruction system provided a student who lacked some knowledge of calculating symbolic fractions with the opportunity to improve their knowledge and skills. The students who attended the remedial course were described as followed. Although six of them answered their state before the remedial course as "they made many mistakes in their calculation" (Table 3), their teacher evaluated them differently. He found inappropriate operations apparently induced from bug-rules in their answer-sheets, and the personal interview of one student supported his evaluation. They seemed to misjudge their state because they were good at calculating numerical fractions and they did not recognize the importance of reasoning or explaining. Because the WWW-based step-by-step instruction system forces them to select the name of every operation during the calculations, we think that it provides a valuable opportunity to change their calculation method.

The WWW-based step-by-step instruction described in this paper is situated between the interactive

textbook and the interactive exercise in the behavior of the students. The best way to use it would be to combine it with the textbook and the exercises in a system.

5. Conclusion

The remedial education of symbolic fractional calculations using a WWW-based step-by-step instruction system is described. The system requests the students to select one of the operations in a list on the computer display for forwarding a step of the calculation, and calculates only this step instead of the student. Eight students, who were not successful with pencil-and-paper supplemental tests, attended the five-day remedial course using the system.

Students were initially embarrassed that the method of the system forced them to reason. However, as they got used to the method, they learned the calculation method of symbolic fractions. After the remedial course, their calculation skills were remarkably improved.

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