ABSTRACT

Web-based Activities as Pedagogical Models

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The study centred on three grades 5-6 teachers' conceptions of algebra and their teaching practice while using a web-based activity dealing with algebraic relationships. The conceptions of algebra held by the teachers in this study correspond to the typical view of algebra that emerges from research, which sees the teaching of algebra being instrumental rather than relational, with a dominance of symbolic algebra over other representations. It appears that their conceptions of algebra did not include the study of algebraic relationships. The web-based activity may have acted as a pedagogical model for teachers' classroom practice, as teachers using the activity shifted their teaching focus from the learning of isolated concepts and skills to the study of relationships among quantities that vary.

Some research indicates that "our thinking is deeply molded by material devices and socio-technical collectives" (Levy, 1993, p. 10). In fact, a powerful trend in psychology and cognitive science argues that mind and environment may be best treated as a unity (Rosch, 1996). It may be possible that well designed and easily accessible web-based activities that allow students to use and explore mathematical relationships may affect what Levy refers to as the "cognitive ecology" of learning environments (Levy, 1993). It is possible that "new forms of representation change the mathematics being taught" (Confrey, 1996, p. 335). New media not only enable us to express our ideas in new ways – they also affect the ideas we have. "We don't always have ideas and then express them in the medium. We have ideas *with* the medium. Making progress is an episode of materially mediated thinking – reasoning or coming up with a new idea – happens jointly in the mind and in the medium at every stage" (DiSessa, 2000, p. 116). And perhaps more importantly, some research indicates that there may be a spillover effect, where learning to think with material devices affects our thinking even when the devices are not present (Borba & Villarreal, 1998).

Web-based activities are the most recent manifestation of technological tools used in mathematics education. They are unique in their focus on small interactive units, due in large part to the current nature of web-access where large programs are unwieldy and quickly exhaust download capabilities and user patience. The smallness of the interactive units is typically coupled with an interface that does not require a steep learning curve for the user, which is often not the case with other technologically-based activities. Activities based on more comprehensive technological tools, such as *Geometer's SketchPad* or graphing calculators, require that students are familiar with an extensive collection of tool-specific procedures for creating representations of mathematics relationships. Well designed web-based activities enable students to engage in investigations of mathematical relationships without having to spend a lot of time learning how to use the tool that creates the various representations of these relationships. For example, the Maximize Area activity (explorelearning.com, 2000), shown in Figure 1, enables students to investigate multiple representations of the problem of finding the maximum rectangular area for a given perimeter. The focused and user-friendly nature of well-designed web-based activities may make them appropriate as demonstrations and explorations of mathematical concepts and new pedagogical directions, as is the case in the Illuminations.nctm.org web site of NCTM (2001), where web-based activities such as the one in Figure 2 are used to demonstrate the pedagogical directions outlined in the *Principles and Standards for School Mathematics* document (NCTM, 2000). Also, as web-based activities are freely available to students and teachers, and as more and more schools and homes are connected to the web, web-based activities may have an impact on classroom and take-home activities.

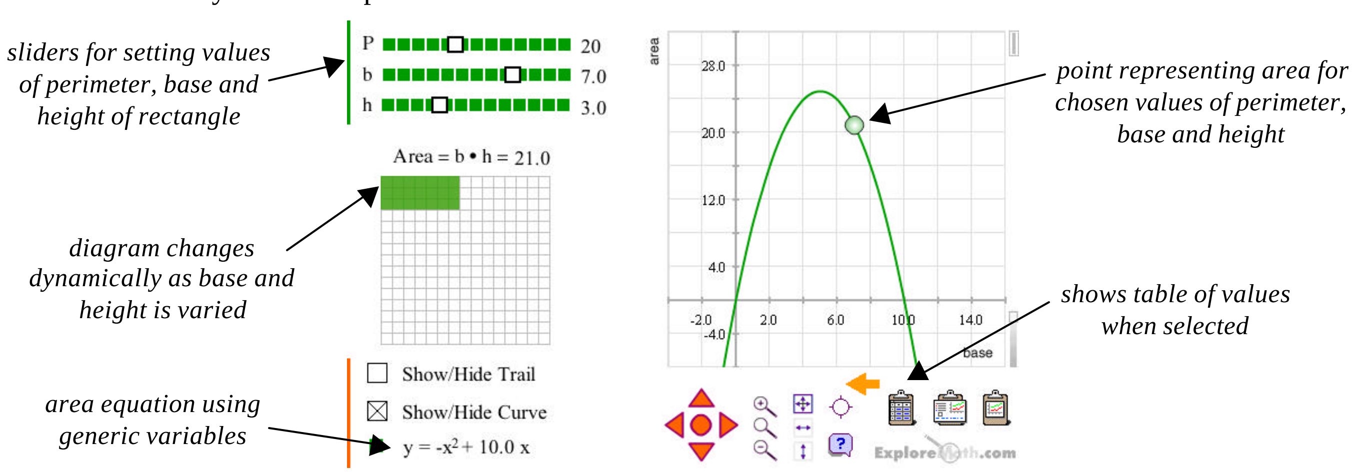

Figure 1. Maximize Area activity.

Web-based activities like the ones shown in Figures 1 and 2 enable students to explore mathematical relationships dynamically. Both activities come with online exploration suggestions for students and lesson plans for teachers. Although the context for each activity involves measurement and geometry concepts, both activities fit well with current algebra curriculum directions (National Council of Teachers of Mathematics [NCTM], 2000, p. 37; Ontario Ministry of Education, 1997, p. 52) as they emphasize relationships among quantities that vary, provide for the investigation of these relationships using multiple representations, and focus student attention on the mathematical analysis of change.

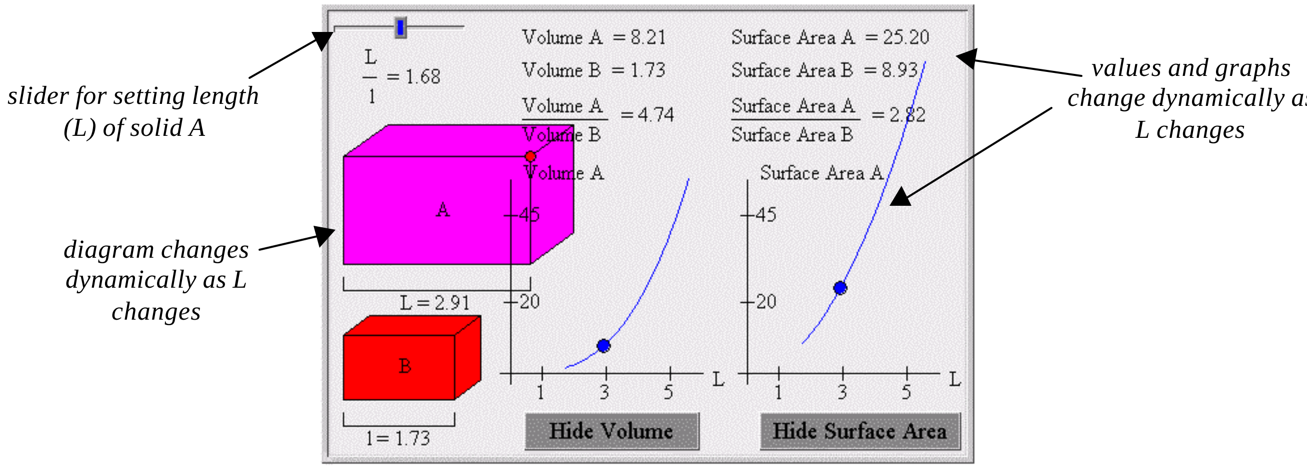

Figure 2. Side Length, Volume, and Surface Area of Similar Solids activity.

The teaching of algebra

The overall conclusion that emerges from research is that the teaching of algebra is typically instrumental rather than relational, with a dominance of symbolic algebra over other representations (Kieran, 1992; Borba & Confrey, 1996; Kieran & Sfard, 1999). Teachers seem to "hold a symbol precedence view of student mathematical development" and they seem to "overestimate the accessibility of symbol-based representations and procedures for students' learning introductory algebra" (Nathan & Koedinger, 2000, p. 209). Consequently, though they learn to manipulate algebraic expressions, students do not seem to be able to use them as tools for meaningful mathematical communication (Kieran & Sfard, 1999). The majority of students do not acquire any real sense of algebra and, early on in their learning of algebra, give up trying to understand algebra and resort to memorizing rules and procedures (Kieran, 1992).

The rules of algebra may appear arbitrary to many students because "all too often they are unable to see the mathematical objects to which these rules are supposed to refer" (Kieran & Sfard, 1999, p. 2). Suggestions for providing students with meaningful experiences in algebra learning include student exploration of multiple representations of algebraic relationships (Borba & Confrey, 1996, pp. 319-320; Kieran & Sfard, 1999, p. 3). It is also suggested that the traditional approach to teaching algebra, which typically starts with symbolic representation and decontextualized manipulation and later moves to visual and graphical representation and problem-based contexts, should be reversed (Borba & Confrey, 1996, pp. 319-320). Graphs, which are often treated as a mere add-on to algebra could become the foundation of algebra teaching and learning (Kieran & Sfard, 1999, p. 3).

"The power of the unaided mind is highly overrated" and representations may be seen as "aids that enhance cognitive abilities" (Norman, 1993, p. 43). It is possible that computer-based representations "can make abstract concepts concrete and manipulative, reveal their properties and constraints, relate them to everyday situations they represent, and connect them to other representations of the same information" (Vosniadou, 1996, p. 22). Also, "while physical objects become more abstract when modeled onscreen (e.g. science simulations), mathematical objects, already inherently abstract, become more concrete" (Lester, 2000, p. 2). Such cognitive tools may reduce cognitive load by taking over some of the more mundane elements of a task (Kieran, Boileau, & Garancon 1996, p. 259; Lajoie, 1993; Surgue, 2000, p. 134) and thus help support students' higher order thinking and hypothesis testing.

The study

The objective of the study is to describe and understand teachers' conceptions of algebra and their teaching practice in the context of using a web-based activity focusing on algebraic relationships. The study focuses on one grade 5 teacher and two grade 6 teachers in a K-8 school in a middle to upper middle class neighbourhood. The school is viewed as an academic school and it has a very high performance rating in provincial testing. The school is located in the province of Ontario, Canada, where Patterning and Algebra is one of the five strands of the curriculum for grades K-8 (Ontario Ministry of Education, 1997). In terms of algebra, the study focuses on grades 5-6 as this is when students start experiencing functions and the symbolic nature of algebra, shifting "from exploring patterns to exploring functions" in grades 5-6 and to "understanding how the language of algebra can be used to generalize a pattern or a relationship" in grades 7-8 (Ontario Ministry of Education, 1997, p. 52). Also, there has been a curriculum trend to move more algebra content from secondary to elementary curricula. For example, NCTM suggests "much of what has been algebra 1 in secondary schools as expected content in the middle grades" (Lott, 2001, p. v).

A case study method is used to report on the pedagogical thinking and practice of teachers as this is appropriate for telling in-depth stories of teaching and learning experiences. The case study method is also appropriate for studying a 'bounded system' (that is, each teacher's thoughts and actions relating to their use of a web-based interactive activity) so as to understand it as it functions under natural conditions (Stake, 1988, p. 255). The analysis of the study is qualitative, as is much of the study on teacher thinking (Thompson, 1992, p. 131) and case studies in general (Stake, 1988, p. 256; Yarger & Smith, 1990, p. 31).

At the beginning of the study, teachers were released from their teaching duties for one day. In the morning, following an overview of the project and the schedule for the day, they shared and discussed their conceptions of algebra and their approaches to teaching algebra concepts, for approximately ninety minutes. The purpose of this was to define a context for subsequent teacher thoughts and actions when using the web-based activities. The following questions/prompts were used: What is "algebra"? How would you define it? How do you typically teach algebra concepts? What types of activities are students involved in? What types of problems do they solve? This led to further clarification questions and discussion. The discussions were recorded and transcribed.

Research on teachers' thoughts and actions suggests that, generally, teachers' beliefs affect practice (Ball, 1988; Cuban, 1988; Madsen-Nason, 1988; Shaw, 1989; Shaw, Davis, & McCarty, 1991; Raymond, 1997). However, research also indicates that there are inconsistencies between teachers' beliefs about mathematics teaching and practice (Thompson, 1984, 1992). Such inconsistencies are accounted for in Green's (1971) model for belief structures which sees no difficulty in teachers holding incompatible beliefs, "provided the beliefs are never set side by side and their inconsistency revealed" (p. 47). Kaplan (1991) suggests that a distinction should be made between *surface* and *deep* beliefs and between *superficial* classroom structures and *pervasive* classroom behaviours. Surface beliefs are more easily modified to accommodate new ideas, and that this sometimes gives teachers "the mistaken impression that they have made significant adaptation in their standard curriculum and teaching methods" (p. 124).

In analyzing transcripts, deep beliefs were identified when there appeared to be a match between the beliefs expressed by teachers and the classroom practice they described. As teachers expressed their beliefs and described their classroom practice in the presence of their colleagues, who have some knowledge of their teaching, it is expected that their statements would more likely be fairly accurate descriptions of their teaching thoughts and actions. The teachers had a good working relationship with one another and often dropped into each other's classrooms. The present researcher also has had previous experiences working with some of the students of the teachers involved in the study, in providing enrichment and contest preparation sessions during regular school hours.

For the last hour of the morning of the day that teachers were released from their teaching duties, teachers were introduced to web-based activities at the exploremath.com and illuminations.nctm.org web sites. In the afternoon, teachers were given the opportunity to explore the various web-based math activities. During the last part of the afternoon, teachers shared their impressions of the web-based activities and their plans for using them in their teaching. The purpose of the day was to expose teachers to a variety of web-based activities and related resources, such as online lesson plans, and allow them to make personal decisions on which activities they would use in their teaching and how they would use them. At the end of the day, the three teachers decided to cooperate in designing a common classroom lesson around the Maximize Area activity discussed earlier as a measurement unit was scheduled in all three classrooms in the following few weeks. They also liked the clear and consistent interface of the exploremath.com activities which are "visually attractive and well laid out [… with] standardized tools for panning, zooming, saving, etc" (Lester, 2000, p. 4).

The teachers were observed as they planned their lessons during after school meetings, and some of the classes were observed as they implemented their lessons. Teachers were released for half a day of their teaching to share and discuss, as a whole group, their experiences and thoughts in using the web-based activity. Meetings were recorded and transcribed, and the transcripts were shared with the teachers, forming the basis for further discussion and elaboration. Follow-up questions were addressed in subsequent lunch meetings. Teacher beliefs about algebra teaching and learning before and after using the web-based activities were compared. Classroom practice was analyzed by examining the various experiences of the teachers when using the web-based activities, with the purpose of identifying common elements that appeared to affect teacher thoughts and actions.

In writing and analyzing the case studies of the six teachers involved in the study, the present researcher made a conscious effort to be objective. The reader should nonetheless be aware of some pertinent experiences and characteristics of the researcher. The researcher has a keen interest in developing and researching web-based activities in mathematics and in developing distance learning courses and resources. In learning and using new software for developing web sites and web-based activities, the researcher has had positive experiences with "materially mediated thinking" (diSessa, 2000) and has noticed that material devices affect thinking even when the devices are not present (Borba & Villarreal, 1998). The researcher also has a deep belief in teachers as agents of change and doubts the effectiveness of top-down reform initiatives – this is a result of previous research of teacher reform visions and practice, and experiences in designing and implementing system wide, computer based reform initiatives.

Teachers' conceptions of algebra

In grades 5-6, the Ontario algebra curriculum deals in part with number patterning, where students identify, extend, and create patterns using sets of numbers, tables of values and graphs. The goal is for students to develop an informal sense of the concept of function and the analysis of change by building "mathematical models to predict the behaviour of real-world phenomena that exhibit observed patterns" (Ontario Ministry of Education, 1997, p. 52). Students are also expected to find missing terms or factors in simple formulas using guess-and-check methods. Problem solving is a focus throughout all strands and grades.

The teachers in the study suggested that there is not a lot of algebra in grades 5-6. They talked about simple patterning, looking at a set of numbers and seeing "how you go from one number to the other." They also discussed solving simple equations through trial and error. One of the grade 6 teachers shared an experience of trying to teach students how to solve simple equations more formally:

> I was teaching grade 6, a sharp bunch. There was hardly anything in the textbook on algebra […] So I went from a math book I had from grade 7 and 8 and I took the simplest pages […] $a – 7 = 10$ and I tried to teach them $a – 7 + 7 = 10 + 7$ and what you do to one side you do to the other side, the way I was taught algebra […] Well they went crazy […] everyday they were like fighting me about the way I was doing it, they wanted to do it in their heads […] they thought I was inventing it, making it up. I said, no, this is the proper way to do it, but I really started to doubt myself.

At the time of the study, the current Ontario curriculum document had been in effect for three years, and the view of algebra it outlines closely parallels the views of the preceding curriculum document (Ontario Ministry of Education, 1993). It is interesting that the word 'modeling,' or other phrases with similar meanings found in the Ontario curriculum descriptions of algebra (Ontario Ministry of Education, 1993; 1997), were not used by the teachers when discussing their conceptions of algebra. And, there was not a sense that the teachers viewed the exploration of algebraic relationships and the analysis of change as integral components of algebra, as is the case in the Ontario curriculum document. What is also interesting is that the teachers' focus on the formal solution of equations is not part of the Ontario curriculum in grades 5-6, nor is it an explicit expectation in the grades 7-8 curriculum (Ontario Ministry of Education, 1997, p. 60). The teachers' view of algebra corresponds to the typical view of algebra that emerges from research, where the teaching of algebra is instrumental rather than relational, with a dominance of symbolic algebra over other representations (Kieran, 1992; Borba & Confrey, 1996; Kieran & Sfard, 1999).

Usiskin (1988) identified four different conceptions of algebra based on the use of variables and the related tasks for students (see Table 1). A similar set of conceptions is identified by Bednarz, Kieran, & Lee (1996, p. 4) as being held by the mathematics community. It appears that the conceptions of algebra held by the teachers in this study included deep beliefs of algebra as generalized arithmetic, as a means to solve certain problems, and as a structure. It also appears that their conceptions of algebra do not include the study of algebraic relationships.

Conception	Use of variables	Student tasks
generalized arithmetic	pattern generalizers	translate, generalize
means to solve certain problems	unknowns, constants	solve, simplify
study of relationships	arguments, parameters	relate, graph
structure	arbitrary marks on paper	manipulate, justify

Table 1. Four different conceptions of algebra.

Using the Maximize Area activity

The teachers in the study used the Maximize Area activity in the context of teaching a unit on measurement. The teachers met and decided on a general direction for the classroom and online activities. The grade 5 teacher then used these ideas to develop a detailed lesson that was shared with the grade 6 teachers. The lesson plan started with the following problem, which was adapted from the online student explorations that accompany the Maximize Area activity: "We're going to build a pen for your dog. Your parents are going to go out and buy 24 metres of fence. You're going to build it with four sides. How are you going to get the biggest area for your dog to play?"

In the grade 5 classroom, the teacher drew one possible pen configuration on the board and then asked the students to find and draw all possible configurations on grid paper. The grade 5 teacher reflected that students "did these without a problem. […] they even understood that not only can we have 11 by 1, we can have 1 by 11. So the dimensions changed but the perimeter stayed constant." Then students developed a table of values, recording the dimensions of the pens they had drawn. They didn't record area values at first "because I didn't want them to try and just rush to an answer […] Then I had them make a third column and do area. When we did this they realized oh the area is changing. So they grasped that […] really well actually."

Then students plotted area versus length on a graph. Students had previous experiences plotting straight line graphs and were surprised that area graph was curved. "And they said oh that's kind of a neat shape because they're used to […] getting a straight line […] and the reason I wanted to do that [in class] was because once they go to the computer that is what they are going to see. […] I didn't want them to go to the computer right away because […] I think they would see the graphs and say that's neat but not relate it to any meaning […] I think that was a big key for their understanding.” As a homework assignment, students were asked to repeat the activity for the case where the perimeter is 20 metres. Students were also told that they would be going to the computer lab the next day to check their answers using a computer program. “I kind of turned it into a game format by telling them ‘you’re going to find out if your answers are right’.” The grade 5 teacher did this on purpose because many of the math computer experiences that students had up to this point had been game oriented.

When the grade 5 students went to the computer lab, they were quickly introduced to the features of the Maximize Area web-based activity, with the teacher drawing parallels with the work that students had done in class and for homework in drawing diagrams, finding values, and creating graphs. Students were asked to set the perimeter to 20 metres and check the diagrams, table of values, and graph that they had done for homework. “They liked seeing this, they liked seeing the area changing.” The teacher was impressed with how well students understood and solved the problem of finding the pen with maximum area. The teacher was also surprised “because I have some grade fives that are not top grade fives but all of them got it.” Some students asked “is the biggest area always a square? […] wouldn’t a circle be bigger? […] they were really thinking.”

When the grade 5 students returned to the classroom after their computer lab experience, they were asked to consider the case where the area is set and the perimeter varies. “Let’s say your dog is now going to be given 36 m^2 of area to play in but the problem is fencing is a lot of money. How can you buy enough fencing to get this area but save as much as you can?” The teacher was surprised that “they got that really quickly on their own. […] I was surprised because I have some grade fives that are weak, but they all got it.”

In contrast, the grade 6 teachers decided to do the computer lab component of the lesson prepared before they did the classroom component. They did this because of their minimal access to the computer lab, and because of the tight timelines of other classroom activities. The grade 6 teachers felt “crunched for time, because if it’s not grade 6 (provincial) testing, it’s VIP, or something else that we’re always being asked to do.” All grade 6 students in Ontario are tested annually in mathematics, and the test scores for schools and school districts are publicized and compared. Also, teachers are required to report on at least three of the five math strands in each of three reporting cycles. By the time the computer lab was available, “we were done the (measurement) unit and felt the pressure to go on to the next report card strand.”

In the computer lab, the grade 6 teachers noticed that some students were not focusing on the problem at hand when they were using the Maximize Area activity. “Some of them thought it was a toy … changing sliders all over the place.” They also noticed that students had questions about the meaning of b and h, as the teachers used length and width rather than base and height in previous lessons on area and perimeter. The grade 6 teachers felt that a little more that half of the students worked well through the activity, used it for the purpose they had intended, and understood the relationships involved.

The grade 6 teachers stated that they felt students going to the computer lab were expecting something different, “it seems they were expecting a game or a fun activity.” The grade 6 teachers noted that the activity worked better with the grade 5 students because “your kids knew what to expect when they went to the lab because they had done those kind of activities (in the classroom), they knew what the problem was all about." The grade 6 teachers said that next time they would reverse the order of the lesson activities, to match the sequence used by the grade 5 teacher. The classroom component of the lesson worked well with the grade 6 students and the grade 6 teachers felt that students learned a lot about the relationship between area and perimeter. After the activity, even the weakest kids, when given certain perimeters and asked to come up with all the different configurations possible, “they were just booming them off.”

The grade 5 teacher apparently experienced greater pedagogical success than the grade 6 teachers as a result of involving students in classroom activities that prepared them for the relationships focus of the Maximize Area activity. It is possible that the grade 5 teacher, having been more involved in preparing the lesson plan, felt a greater sense of ownership and consequently a greater level of commitment to the lesson plan sequence than the grade 6 teachers.

The Maximize Area activity may have acted as a pedagogical model for teachers' classroom practice as the approach to teaching about area and perimeter described above varies from the norm. In previous years, teachers did not focus on the relationships between area and perimeter. They did use tables of values involving area and/or perimeter where students had to find missing values, however, “I did tables and graphs but not necessarily with the same perimeter to find the area […] The length and width were changing, the area was changing, it wasn’t constant perimeter or constant area […] nothing that had a relationship.” The typical approach to area and perimeter is more reliant on the development and use of formulas, with minimal emphasis on the relationship between area and perimeter. The grade 5 teacher said, “I think I do more the traditional way, separating the two.” One of the grade 6 teachers said, “I do perimeter first.” The grade 5 teacher reflected on the effect of this difference on students: “When I introduced area and perimeter [in previous years] they didn’t see a relationship between the two, but when you fix one at a given constant, and you can manipulate the other, they really found that pretty neat, especially when you apply it to something outside of math [like the dog pen problem].”

Discussion

Teachers’ conceptions of algebra

When asked if algebra was involved in the student classroom and online activities relating to the Maximize Area activity, the teachers talked about the area equation, and the substitution of values in the equation. “A mystery, an unknown, that’s what I see. That’s algebra, right?” Another teacher described algebra as “numbers and unknown letters, solving by doing the same to both sides.” When asked if the relationships between area and perimeter and the representation of these relationships using diagrams, tables of values, and graphs that are an integral part of the Maximize Area activity were also part of algebra, teachers said they didn’t think so. One teacher said that some algebra was involved in creating a table of values, as substitution in an equation was used to find values. The teachers recognized the increased focus on relationships in the Maximize Area activity, compared to their regular teaching. However, they did not associate this relationship focus with algebra, even though that connection is in the Ontario mathematics curriculum document. The only aspects of algebra they recognized in the Maximize Area activity was the equation for area, and the substitution in this equation to generate a table of values. One may wonder whether this makes a difference in teaching mathematics – that is, as long as teachers focus on algebraic relationships when they teach about area and perimeter, does it matter that they do not see them as algebra? This diminished view of algebra makes a difference for at least two reasons. First, algebra is typically a strand of mathematics in curriculum documents. The focus on relationships helps integrate and relate the mathematics strands that students study, as is the case with the Maximize Area activity which integrates measurement and algebra. Isolating algebra to a symbolic and manipulative category reduces its richness as a topic of study. Second, many teachers view area and perimeter as an elementary school topic of study and algebra as a secondary school topic of study. As elementary teachers naturally try to prepare their students for success in secondary school mathematics, the richness of their view of algebra will likely make a difference in what algebra they teach and how they teach it.

Further research on teachers' use of web-based activities may explore their perceptions of curriculum documents and the relationship between curriculum directions and teachers' conceptions of algebra. As web-based activities may be used to model new curriculum directions, as is the case at NCTM's illuminations.nctm.org web site, research may also explore the role that web-based activities may play in the education and professional development of mathematics teachers. As the teaching of algebra is typically instrumental rather than relational (Kieran, 1992; Borba & Confrey, 1996; Kieran & Sfard, 1999), web-based activities such as the Maximize Area activity (Figure 1) and the Side Length, Volume, and Surface Area of Similar Solids activity (Figure 2) may be used as models in the education and professional development of mathematics teachers.

Using the Maximize Area activity

The study involved one web-based activity used on a limited basis by three teachers. The study offers some support for existing research that shows that material resources affect the mathematics taught (Levy, 1993; Confrey, 1996; diSessa, 2000). The Maximize Area activity's focus on algebraic relationships and on multiple representations of such relationships seems to have had some effect on shifting the mathematics teaching focus from isolated concepts to relationships among concepts. However, the specific effects in each of the classrooms, including the level of pedagogical success experienced by the teachers, varied. The Maximize Area activity was used differently by the grade 5 teacher and the grade 6 teachers, even though all teachers collaborated in developing a common lesson plan. It is conceivable that such differences may be greater for teachers working independently of one another. The experiences of the three teachers using the Maximize Area activity suggest a "cognitive ecology" (Levy, 1993; Rosch, 1996) among various elements in the mathematics teaching landscape, which includes the nature of the web-based activity, the conceptions of the teacher and the students, as well as other environmental factors such as curriculum and technology constraints. Further studies of teachers using similar web-based activities could clarify the effect of web-based activities on the teaching of mathematics and on the relationships among existing and new elements in the mathematics teaching landscape.

It would be interesting to consider subsequent teaching practices of teachers to explore whether there is a spillover effect, that is, whether thinking pedagogically in the presence of web-based activities would affect teacher thinking even when the web-based activities are not present (Borba & Villarreal, 1998). Research also needs to focus on the thinking of students and how students engage mathematically when using such activities. It appears that the likelihood of pedagogical success in using web-based activities may be increased if students are initially involved in classroom activities that prepare them for the types of relationships that they would explore using web-based activities.

Web-based activities, such as the Maximize Area activity, are unique compared to most computer-based activities, as they are accessible in new ways, they tend to be narrowly focused, and are designed to be easy to use. Further research may explore the effect of these factors on how teachers and students use them. Is it possible that the nature of take-home assignments may become more investigation-based as opposed to practice-based?

Conclusion

We could make the argument that the effect of technologically based tools depends greatly on how they are used. We could also make the argument that how such tools are used depends on the nature of the tools themselves. The study indicates that web-based activities, such as the Maximize Area activity (explorelearning.com, 2000) and the Side Length, Volume, and Surface Area of Similar Solids activity (NCTM, 2001), may act as pedagogical models for teachers' classroom practice. They have the potential of capturing the essence of recommended algebra curriculum reform (NCTM, 2000; Ontario Ministry of Education, 1997) and of related research-based suggestions (Kieran, 1992; Borba & Confrey, 1996; Kieran & Sfard, 1999). The study also indicates that the effect of such activities is a product of complex relationships among various elements in the mathematics teaching landscape.

Algebra may be viewed as a language for representing and exploring mathematical relationships. Current curricular views of algebra emphasize multiple representations of algebraic relationships among quantities that vary, and a focus of student attention on the mathematical analysis of change in these relationships (Ontario Ministry of Education, 1997, p. 52; NCTM, 2000, p. 37). The relationships component of such a definition of algebra is what gives purpose and meaning to the language aspect of algebra. Without a focus on relationships, the language of algebra loses its richness and withers to a set of grammatical rules and structures.

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