Improving Student Understanding About Computational Processes, Problem Solving Techniques, and Proof Using Hand-Held Technology

Presented by

Franklin Demana
The Ohio State University
Columbus, OH 43210
Email: demanaf@math.ohio-state.edu

This paper was written in collaboration with Bert K. Waits, Professor Emeritus of Mathematics, The Ohio State University.

Background

After a little more than two decades of numerous pioneering efforts in the U. S., we believe that we can safely say that the use of hand held technology has forever changed the way mathematics is taught, and forever changed the way students learn mathematics. Our philosophy about using technology in instruction developed through the experience gained in our work in the Calculator and Computer Precalculus (C^2PC) project (Waits & Demana, 1994) that started in 1985. The C^2PC project extended the work of Demana, Joan Leitzel, A. Osborne, and J. Crosswhite in the Transitions to College Mathematics project that started at The Ohio State University (OSU) in 1980 (Demana & Leitzel, 1988). The Transitions project was expanded to include middle school mathematics in 1983 and called the Approaching Algebra Numerically (AAN) project (Comstock & Demana, 1987). The Transitions, AAN, and C^2PC projects grew out of the OSU effort to reform the college remedial mathematics curriculum that began in 1974 and required the use of four-function calculators by all students (Waits & Leitzel, 1976).

Soon after the C^2PC project started, the Sloan Conference was held at Tulane University in January, 1986, and sparked the calculus reform movement in the U.S. (MAA, 1986). This movement was fueled by the National Science Foundation as it provided millions of dollars in grants for calculus reform shortly after the national conference “Calculus for a New Century” was held in Washington, D.C. in October, 1987 (MAA, 1987) which we attended.

A Balanced Approach to Curriculum Reform

Prior to the advent of easy to use hand held technology, about 85% of the mathematics curriculum consisted of paper-and-pencil computation. The computation involved the algebraic and analytic process of mathematics including the common symbolic manipulations of algebra and calculus (by paper and pencil). This type of computation usually involved very low order thinking skills, and often has been associated with the phrase “drill and kill mindless manipulations.” In the pre hand held technology curriculum, there were precious few application examples and they almost always occurred as consequences of mathematics concepts developed algebraically or analytically. Further, little or no real proof occurred in the standard courses. There is growing evidence that paper and pencil manipulation skill alone does not lead to better understanding of mathematical concepts. Indeed, the appropriate use of hand held technology could provide more classroom time for the development of better understanding of mathematical concepts by eliminating the time spent on “mindless paper and pencil manipulations.”

The advent of affordable scientific calculators and graphing calculators allowed students to make widespread use of numerical and graphical techniques. Suddenly, students were able to make regular use of numerical and graphical problem solving techniques. Most educators believe that technology, problem solving, and the use of real data should have an expanded role in the current mathematics curriculum. Here is an example of how real data and graphing calculators can change lessons about percent.

Example 1 Table 1 shows the initial seed position of the eventual NCAA men's basketball champion from 1979 to present. Figure 1 shows this categorical data entered into lists L1 and L2 of the new Texas Instruments middle school graphing calculator (TI-73).

Table 1 Men's NCAA Tournament

Seed Position	Winners
1 st	9
2 nd	4
3 rd	2
4 th	1
6 th	2
8 th	1

Source: NCAA as reported in *USA Today* on March 9, 1998.

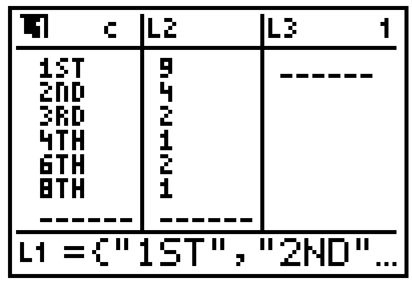

Figure 1 The "c" in column 1 indicates that the list contains categorical data.

Then Figure 2 sets up a circle graph representation on the data in Table 1 in percent form. Notice that the category "1st" (L1) has frequency 9 (L2) and represents about 47.368% of the 19 years from 1979 through 1997. This means that about 47.368% of the NCAA men's basketball champions were teams seeded 1st in one of the four regions during this time period. We can also read that slightly more than 68% (about 2/3) of the eventual champions were seeded either 1st or 2nd.

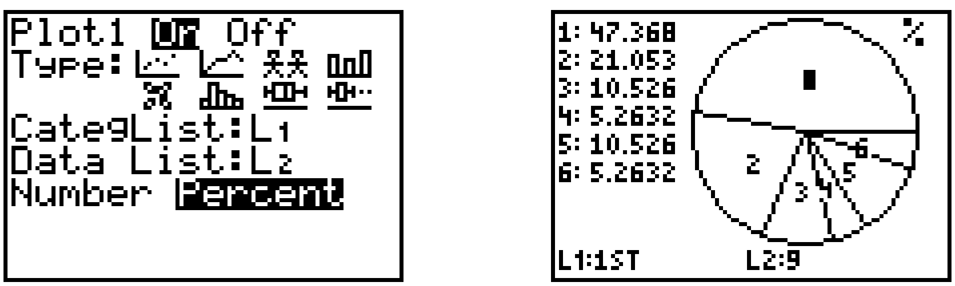

Figure 2 A pie chart representation of the data in Table 1.

Figure 3 suggests important components of a modern balanced curriculum. We expect that problem solving and proof (or giving convincing arguments) will play a more important role, and paper-and-pencil computation will occupy a smaller share of a balanced modern curriculum. We do not mean to suggest that the time spent on these features should be the same. However, computation should not take up 85% of class time as it did in the past.

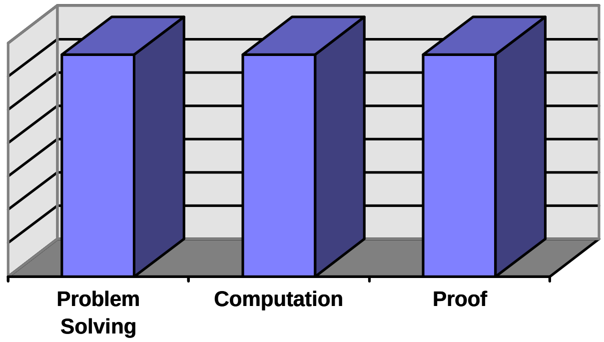

Figure 3

Many mathematicians are becoming increasingly concerned with the perceived lack of attention to paper-and-paper skills in the new evolving mathematics reform curricula. There have been numerous articles in the “Notices of the American Mathematics Society” during 1996 and 1997 giving pro and con views about the reform effort and the use of technology. Robin Wilson (February 1997) gives a glimpse of the division among mathematicians on “reform calculus” in his article in the Chronicle of Higher Education. Speaking against reform calculus:

> “This approach really shies away from anything but superficial use of skills,” says Ralph L. Cohen, a professor of mathematics at Stanford University, which after seven years has decided to stop teaching the ‘reform calculus’ and to move back to something more traditional. “For students who really need to know math and use it, this wasn’t nearly sophisticated or rigorous enough.”

Describing the issues involved in the debate:

> “The debates are as deep as those between two different religious groups,” says Ronald G. Douglas, provost at Texas A&M University, who is considered the father of the reform movement.

Speaking in favor of the reform movement:

> “They (students using traditional methods) learned they could stick in a couple of key symbols, statements, and equations and put forward what were found to be acceptable solutions, even though they had no idea what was going on,” says Morton Brown, a professor of mathematics at the University of Michigan.

In our opinion much of the debate revolves around misconceptions on both sides about the goals of the reform effort. In our opinion, it is not the goal of the reform effort to abandon algebraic or analytic techniques. Yet teachers sometimes give this impression or mistakenly believe that this is true in their zealous advocacy for the use of technology. For example, the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) clearly states that certain algebraic techniques should receive decreased attention in the curriculum. Some teachers wrongly infer that if less attention is good, then no attention is better. A careful reading of the Standards shows that this was never the intention of the authors.

Actually, paper-and-pencil skills are and will continue to be an important part of the curriculum. However, the role of paper and pencil computation will change dramatically in the future because of hand-held technology. Technology provides a “better mouse trap” for much computation. We must recognize and exploit this fact.

Our new challenge is to think about computation differently. Each paper-and-pencil algorithm should be analyzed to see if the procedure contributes any understanding to the process. If not, it should be removed and performed with technology. For example, there is probably widespread agreement that the square root algorithm and finding trigonometric and logarithmic values from a table by interpolation are obsolete. The concept of interpolation is not obsolete, as it is an important idea in mathematics. Using interpolation to find values of trigonometric and logarithmic functions from a table is obsolete. Hand held computer symbolic algebra (CSA) will soon make many of the paper-and-pencil factoring algorithms obsolete but not the process of factoring (which is a key concept in the fundamental theorem of algebra). The same will be true for many of the paper-and-pencil symbolic procedures typically taught today.

We believe computation should be done in one of the following three ways today and in the future. (By computation we mean those manipulative procedures associated with paper-and-pencil arithmetic, algebra, and calculus.)

1. Mental computation
2. Paper-and-pencil computation
3. Computation done with technology

Our challenge is to decide when a given computation method is appropriate. We believe that some computations will be judged to be mental or paper-and-pencil computation in one course (or section of a course), but then should be done with technology in subsequent courses (or section of the course). This pedagogical technique is called the *white box/black box* principle. For example, partial fraction decomposition in calculus is a “black box” procedure best done with technology. But integration of functions is a white box procedure (using paper and pencil). That is, we allow the use of some algebraic, non-calculus, black box procedures while not allowing any black box integration procedures (until the skill or concept is learned). Professor Bruno Buchberger, Research Institute for Symbolic Computation in Linz, Austria, first introduced the white-box/black box principal. This wonderful principle is outlined in detail along with other excellent examples in the book by Heugl, Klinger, and Lechner (Addison Wesley, 1996).

We also need to analyze paper-and-pencil procedures to see if technology can add understanding about the underlying concepts. The following examples illustrate that technology seems to deepen student understanding about some paper-and-pencil processes.

Example 2 Figure 4 shows the prime factorization of 10! = 3,628,800 using the TI-92. We can now ask our students to confirm this factorization using mental computation, by counting the number of times each prime appears in 10! With paper-and-pencil alone we would not likely ask our students to factor such a large number. This mental activity deepens student understanding about factorization.

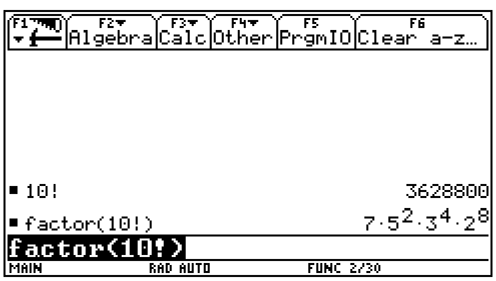

Figure 4 Notice that $10! = 3{,}628{,}800 = 7 \cdot 5^2 \cdot 3^4 \cdot 2^8$

Example 3 Solve the system of equations:

$$3x + 4y = 5$$
$$2x - 3y = 4$$

Solution: We use the method of substitution, an important mathematical process, to solve for *x* and *y*. The first line of Figure 5 shows how to solve the first equation for *y* in terms of *x* using the TI-92. The second line shows one way to define *y* as a function of *x* on the TI-92, a step that is not very well understood by many students when they use paper-and-pencil techniques to solve the system. Understanding this TI-92 step contributes to student understanding about the process.

In the third line, we solve the second equation for *x*. Since we have defined *y* to be a function of *x* we should expect to get a number. Again deeper understanding is possible for students. The fourth and fifth lines obtain the values of *x* and *y* that solve the system.

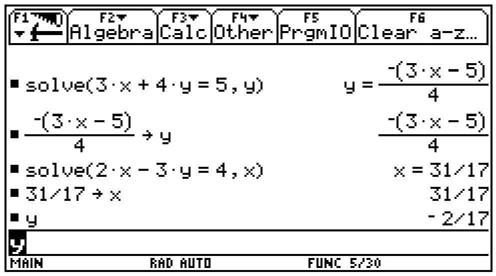

Figure 5 Solving a system of equation using the TI-92.

All students can then use this same procedure to solve a general system of two linear equations in two variables and discover Cramer's Rule. This type of computation does not usually occur in a non-technology paper-and-pencil environment.

Proof, or giving convincing arguments, is another area of the curriculum that can be expanded through the use of technology. There is precious little of this in a standard mathematics curriculum except possibly in geometry courses. Even in most geometry courses, students are really only committing facts to memory that are later spewed out on examinations.

The ability to obtain many correct symbolic results is an important feature of technology. Students can obtain many correct statements quickly to help them form correct generalizations. This type of algebraic exploration is not possible for many students using only paper-and-pencil techniques. However, with technology, all students can experience symbolic exploration. For example, Figure 6 shows three cases of the following identity using the TI-92 computer algebra features.

$$\sin(nx) + \cos(nx) = \sqrt{2}\sin(nx + p\ /\ 4)$$

Now, with teacher encouragement and guidance, students could use the sine of the sum of two angles formula to analytically confirm or prove the above identity.

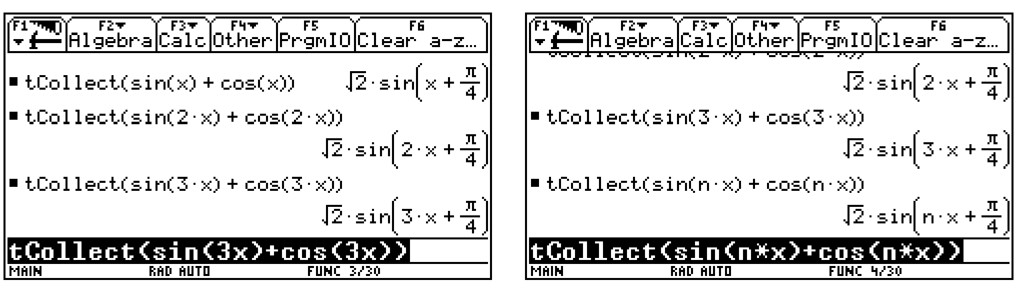

Figure 6 Using the trigonometric computer algebra tCollect(command of the TI-92.

With appropriate use of technology, all students can participant in a balanced mathematical curriculum that has problem solving, computation, and proof as major themes. Appropriate use of scientific calculators and graphing calculators helped empower all students numerically and graphically. Now appropriate use of CAS can empower all students symbolically.

Staff Development

Successful technology based reform requires two important ingredients. First, you need to provide teachers with technology based materials (for example, Demana, Waits, Clemens, & Foley, 1997; Finney, Demana, Waits, & Kennedy, 1998). These two books are considered to be the first textbooks to fully integrate technology in precalculus and calculus. Now most secondary and collegiate algebra, precalculus, and calculus textbooks incorporate technology. Second, teachers need to be trained in the appropriate use of technology to enhance the teaching and learning of mathematics. We found that we needed to develop a massive professional development program for teachers, that is now called the T^3 (Teachers Teaching with Technology) program (Demana & Waits, 1997).

The key to implementing the technology-based approach to teaching mathematics we use is to provide teachers with intensive start-up training and regular follow-up activities. We cannot expect teachers to make fundamental change in their teaching without a good deal of help and ongoing support. This may seem like a hopeless task, but our work shows that it is indeed possible.

The highly successful Teachers Teaching with Technology (T^3) inservice program that we founded can serve as a model for such training. This program was started at Ohio State University. In 1988, the first National Science Foundation (NSF) funded one-week intensive Calculator and Computer Pre Calculus (C^2PC) summer institutes were held at Ohio State University. A total of 80 secondary mathematics teachers from around the U.S. received instruction on how to use graphing calculators and visualization to enhance the teaching and learning of precalculus mathematics. Each teacher returned to their classrooms with an implementation plan that required using graphing calculators or computers with our technology enhanced precalculus materials.

The program expanded rapidly through outside funding. Current institutes are partially supported by Texas Instruments, Inc., as well as local and state funding such as Eisenhower funds. The key to this successful expansion is our cadre of nearly 150 highly trained mathematics and science teacher leaders to deliver the institutes. We have an annual and about six regional T^3 conferences designed to provide ongoing follow-up activities for the teachers trained in the summer institutes.

In 1998, we anticipate holding about 300 T^3 institutes with approximately 9,000 teachers. While these numbers are impressive, they still seem meager when we look at large cities. For example, New York City has about 62,000 teachers and about 1,000,000 students.

The key to successful technology reform in mathematics teaching and learning, indeed to any reform, is a massive network of teacher leaders capable of providing appropriate training to all teachers in a relatively short period of time.

Impact on Teaching and Learning

Scientific calculators are now very inexpensive ($10 to $20) and have significantly changed the mathematics curriculum taught in most countries. For example, we no longer spend valuable lecture time teaching paper and pencil methods to evaluate transcendental functions. More time is spent on applications and conceptual understanding of transcendental functions as scientific calculator use has become widespread. Desktop computers have remained expensive and thus still are not used nearly as widely as they should be in the teaching and learning of mathematics in colleges and universities.

Initially, we, as well as many others involved in the modern reform effort, underestimated the impact on teachers of using technology to enhance the teaching and learning of mathematics. We naively referred to this change as an incremental approach to reform. However, the use of technology requires teachers to completely change the way they teach. Teachers also need to be prepared for the complete change in the way students learn using a technological approach. We discovered that most teachers found this change to be revolutionary and not the least bit evolutionary or incremental.

We observed that surprising few of our students were able to get started on a problem using symbolic techniques. However, virtually all of our students were able to begin the analysis of a problem using numerical or graphical techniques. The numerical or graphical investigation laid the foundation for students to represent and then solve problems using algebraic or analytic techniques. Students, even our remedial students at OSU, became more flexible problem solvers. They were able to attempt problems using a variety of techniques, and did not give up when they were not able to solve problems initially using algebraic or analytic techniques.

Prior to the wide spread use of graphing calculators, we observed that students had virtually no understanding about the use of graphing as a tool to do mathematics. For example, even our best calculus students at OSU did not initially know what it meant to solve an equation of the form $f(x)=0$ graphically. Their understanding about graphs was so minimal that creating a graph by hand to solve the equation was an impossibly time consuming task. Now, solving equations and other problems graphically, even in algebra courses, is fairly routine. We regularly ask students to solve problems using technology, and to *confirm* the answer algebraically or analytically using traditional paper-and-pencil techniques. Not only do students develop powerful graphical techniques, but also teachers can focus on the mathematical reasons that the graphical techniques work.

We also ask students to solve problems using traditional paper-and-pencil methods and then *support* their results graphically. For example, we ask students to use analytic techniques to find solutions to optimization problems or to find coordinates of points of inflection. Then we ask them to support their results by graphing f, f', and f'' in the same viewing window. This helps students to establish the connections among these graphs in an incredibly powerful visual way.

One of the more compelling stories about our earlier work in the C^2PC project was with graphs of rational functions. Our project students, both high school and college, give the following naive description of the class of rational functions. They used paper-and-pencil division to express the rational function *f/g* in the form

$$\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)},$$

where *f*, *g*, *q*, and *r* are polynomials with either $r(x)=0$, or the degree of *r* is less than the degree of *g*. Then, they described the construction of the graph of *f/g* as follows: First draw the graph of the polynomial *q*, and then erase a small portion of the graph near each zero of *g*. Complete the graph using the behavior of the function *f/g* near the zeros of *g*. In their language, the graph of a rational function is basically the graph of a polynomial except for a few bad places.

We have always tried to emphasize balance. Our well-known graphing calculator C^2PC philosophy was a call for balance.

DO algebraically with paper and pencil, and then **SUPPORT** the result with computer graphing,
DO using computer graphing, and then **CONFIRM** the result using paper-and-pencil algebra,
DO using computer technology because this is the only (or only practical) way.

Research Results

We and the teachers working with us in our projects have observed students learning to value mathematics in ways we only dreamed of in the past. The use of real data found in newspapers and other sources, as well as data collected using CBLs and CBRs give students a better feeling about mathematics. We now see how a technology-enhanced approach to teaching can empower all students to participate and achieve in mathematics.

Many mathematics education research studies have been conducted as an outgrowth of the C^2PC project. There is a summary of several of these studies in a paper by Dunham (1992) that appeared in the Proceedings of the "Fourth International Conference on Technology in Collegiate Mathematics" that was founded by Waits and Demana. In fact, there are numerous research reports that appear in the proceedings of this annual conference, the tenth of which was held in 1997.

An excellent source of information about issues related to use of technology in instruction is contained in the book titled, "Impact of Calculators on Mathematics Instruction" (Bright, Waxman, & Williams, 1994). In the concluding chapter of this book the editors describe four specific areas that have important implications for future research in the field:

1. The impact of theory and research from the field of cognitive psychology on the use of calculators in mathematics classrooms.
2. Research studies needed in the area of staff development and training.
3. Issues related to evaluative research studies.
4. Need for more programmatic research in this area.

We recommend that this book be read from cover to cover.

A recent article (Nicol, 1997) describes the process of how one physics teacher changed his attitude about algebraic thinking. It shows how difficult it is for teachers to change the way they teach. This study certainly supports the need for research studies in the area of staff development and training called for by Bright, Waxman, and Williams (1994). Interesting, the teacher in this study is now one of our national T^3 Instructors.

Another important source of information about use of technology is the book titled, "Integrating Research on the Graphical Representation of Functions" (Romberg, Fennema, & Carpenter, 1993). This book is concerned with the integration of research on teaching, learning, curriculum, and assessment with respect to the graphical representation of functions. In his chapter, Jim Kaput argues that research in the representation of quantitative relationships should be further ahead of current practice than it is now in at least three dimensions: technology, curriculum, and representation. This is another book that warrants reading from cover to cover.

The process of change called for by the reform effort goes beyond what researchers and curriculum developers can do by themselves. This is an issue for parents and all of society. Peressini (1997) describes the importance of parental involvement in the reform of mathematics education.

The Future

Graphing calculators, when compared to hand-held CAS like the HP-48, CFX-9970G, TI-89, and the TI-92 do not lead to significant change in core curriculum content. Graphing calculators are wonderful tools that can be used to *enhance, not replace* the current core curriculum. They enable better pedagogy for teaching mathematics, facilitate the incorporation of problem solving activities and applications, provide a motive for asking students to think about mathematical concepts and use them to justify the use of technology, and help students learn to value mathematics. But graphing calculators do not, nor will not, cause significant core curriculum change. The graphing calculator revolution was teacher driven (teachers required their use and purchase) rather than student driven (students purchase them because they see their value).

Hand-held CSA tools will no doubt become the scientific calculator of the future, and will be demanded by all students just like they demanded scientific calculators. Hand-held CSA tools *will* *dramatically change* (replace in many cases) the core mathematics curriculum taught in grades 9-16 in the next 5-25 years in this country and around the world. We do not know yet exactly how we will get there. A great deal of experience and research is needed. A major issue will be how to test with CAS. The controversy about testing in the presence of graphing calculators will be mild compared to testing in the presence of CAS. We need to begin today to plan for this next dramatic step in the teaching and learning of mathematics.

We believe what is needed today and in the future is a school and university mathematics curriculum that takes advantage of computer technology to assist students in gaining mathematical understanding, in becoming powerful and thoughtful “thinkers,” communicators, and problem solvers. We seek a *balanced approach* to the use of technology.

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